Math Problem Solving (MPSE)

CTY Course Syllabus

Day: 1
Objectives:

- Create a positive classroom environment.

- Get preliminary data about student problem solving skills.

- Introduce and practice the four step problem solving process.

Morning

- Introduce myself using the personal shield template.

- Have students complete the written pre-assessment.

- Review CTY honor code, technology form, general rules, procedures, homework policy, and classroom norms.

- Offer the students some preliminary problems such as "There are twelve kids and two adults in this class. How many toes are there in this class? How many ears? How many noses?" Ask the students what they usually do when they are given a problem and they don't immediately know how to solve it. Introduce the four steps of problem solving:

  o Understand what is being asked.

  o Devise a plan to figure it out.

  o Carry out the plan.

  o Look at your answer again to see if it makes sense, and to check over your work.

Afternoon

- Give the students the basketball problem on page 14 of Problem Solving Strategies. Require the students to draw a diagram (perhaps drawing each of the teams by their mascots) to find the answers, working in pairs. After they are done and they've shared their answers have them work on the Model Trains problem (pg 17). Require and then look for diagrams for each of these solutions.

Homework

- Have the students use the four problem solving steps to draw a diagram to answer this question: Leslie has 25 cents in her pocket, but she doesn't have a quarter. How many different combinations of coins could she have that add up to 25 cents? Make sure you have a diagram to show how you got your answer.

Day: 2

Objectives:

- Review Drawing a diagram strategy

- Introduce the idea of making organized lists.

- Introduce basic probability to solve problems.

Morning

- Review the homework. Have students swap papers with a partner and compare answers.

- Carry-over problem from yesterday: Alien Invaders (pg 18).

- Pose this problem: *How many different after school options are there if you can play baseball or basketball first; homework, read or do your chores second; help with dinner or make your bed third; and watch a show, play on your iPad, play on the computer, or listen to music last?*

- Brainstorm ways that we can make a list to find the solutions.

- Do a similar activity but with outfits (different shirts, shoes, pants, etc.). Have the students draw the different possibilities and cut them out. With the cutouts have them create different outfit combinations until they think they've gotten them all.

- Go back to the basketball problem from the previous day. Have the students make an organized list to solve the problem.

Afternoon:

- Have the students first predict how likely it is that if a dice is rolled it will land on a 6. Explain probability regarding the chances of landing on certain numbers. Then, ask what the probability is that it will land on a 6 OR a 4.

- Introduce the students to basic probability vocabulary. Have them predict the outcomes in pairs, playing a game of probability. After they're done, compile all the class results. Talk about why the exact probabilities didn't pan out how we thought they would.

- Discuss experimental probability and theoretical probability.

Homework:

- Have the students use the four problem solving steps to make an organized list to answer this question: A rectangle has an area measuring 120 sq. cm. Its length and width are whole numbers of centimeters (no decimals). List all the possibilities of lengths and widths that I could be. (Page 39).

Day: 3
Objectives:

- Continue to use the four step problem solving technique for a variety of problems.

- Use diagrams, lists, and concepts of probability to solve problems.

- Show the students how to make educated guesses and then check their guesses.

Morning

- Review the homework. Have students swap papers with a partner and compare answers.

- Review the Fundamental Counting Principal by showing tree diagrams. Have students conduct this simulation: You are going to the zoo. You are in a bit of a hurry, and you can only visit one enclosure (cage) at each of the following three locations- Reptile Room, African Safari, Florida wetlands. [These locations will be represented by different areas of the room.] The reptile room has 3 enclosures, the African safari has 4 enclosures, and the wetlands have 3 enclosures. How many ways can you visit just one enclosure for each of the areas at the zoo? Help the students walk through a tree diagram for this problem.

- Have the students create organized lists to show the different possibilities.

- Also go over theoretical and experimental probability with different dice simulations, including finding which sum is the most likely and why when two dice are rolled.

Afternoon

- Present the students with the Farmer Jones problem on page 164. Have them make some guesses and discuss which ones are reasonable, and which ones might not be correct. After going over the problem, ask what made a good guess, and discuss how one guess could help with a second guess.

- Using the draw a diagram idea, have the students do the 'All Around the Playing Field' problem on page 167. They should draw a diagram and then make some educated guesses.

- Do the Zeke and Cloe Reveal Their Ages problem on page 175. Have the students work in groups, then write their own similar problem. Have them switch papers with another group to solve.

Homework:

- Dan has twice as much money in nickels as he does in quarters. He has 33 coins in all (he only has nickels and quarters). How much money does he have?

Make a chart to use with guess and check! (Nickels are 5 cents and quarters are 25 cents each.).

Day: 4

Objectives

- Review the homework. Have students swap papers and try each other's problems.

- Review the guess and check strategy, discussing what makes a good guess and how one guess can guide a second one.

- Introduce and use the Patterns and Models to recognize and solve relationships among shapes.

Morning

- Introduce pattern blocks. Have students work through problems at pattern block problems

- Students will get to work with the blocks and compare them. They will get to create their own designs.

- Have students create different quilts of fourths. See this website for examples: quilts of fourths examples

Afternoon

- Show students the handshake problem at handshake problem

- Have them try it on their own for 10-15 minutes. Discuss what they have been trying. See if anyone recognized a pattern or created a model. Then, have the students show their work on large poster paper. Have them present to the class.

- Discuss the different strategies and which ones worked and which ones didn't.

Homework

- If you take a certain two-digit number and reverse its digits to get another two-digit number, then add these two numbers together, their sum is 132. What is the original number?

Day: 5

Objectives

- Review the Homework.

- Discuss and review the importance of communicating mathematical thoughts and processes through writing and explanation.

- Use the methods already learned- such as pattern recognition and guess and check- to decipher a code.

Morning

- Explain the difference between a code and a cipher. I will show students examples of each. Show students the Caesar's cipher. Discuss different ideas about how we might figure it out. Have student write out, in complete sentences, how they think it can be translated.

- Work through the cipher together.

- Show students some simple computer programs that can translate text very quickly. [Simple computer programs examples](#)

- Have students write their own cipher that must be at least five sentences long.

Afternoon

- Have the students trade and decipher the ciphers. Discuss the probability of different letters as a strategy to solve the problem. Have the kids notice any interesting observations.

Homework

- What are the next three terms in this sequence: 1, 1, 2, 3, 5, 8, …?

Day: 6

Objectives

- Continue using pattern recognition and guess and check to solve codes.

- Use pattern recognition to develop strategies in playing a game.

- Introduce the factor game ([Factor Game](#)). Go over the directions and factor vocabulary with the students.

- Play the factor game discussing different strategies.

- After playing together as a whole group, have the students work on it in small groups. The game board and instructions can be found here: [Factor Game PDF](#)

Afternoon

- Introduce the students to the idea of variables. Complete the lesson at [Variables Lesson](#)

- Have students do the activity, making their own bands.

- Introduce the students to the 'Lost in Translation Project' where students will be deciphering a code and then creating their own. Students will need to WRITE OUT an explanation to cracking their code and write a story about who might use the code. This project will be put on a large poster and worked on throughout the session.

Homework

- Students will write up an outline for the final project. They will need to start brainstorming ideas for a code, start brainstorming ideas for the people who use the code, and how they will lay out the poster.

Day: 7

Objectives:

- Have students use patterns and tables to figure out how many chairs can fit around a table.

- Have students use patterns and tables to solve a simpler problem before trying a more difficult one.

- Investigate sequences and the patterns that make them.

Morning

- Introduce the problem of determining the number of chairs needed in a restaurant with different types of tables. There are many different varieties and adaptations to this problem, as can be seen on the website. See [Chairs around the table problem](#)

- Students will get to actually act out many of the different arrangements for the tables, moving chairs around and investigating different ways that chairs can be arranged around a table.

- The idea of a variable will be reintroduced, but this time it can change. As in, *n* doesn't just equal 4, but *n* can represent an unknown number like n tables.

Afternoon

- Pose the problem of counting squares on a checkerboard. Have the students look at an actual checkerboard. Have them brainstorm different ideas about how many there are. See counting embedded figures for more information and ideas.

- Sequence problems on page 122, 139, and 140.

Homework

- Problem Set 5 from the book (Page 151).

Day: 8

Objectives:

- Review Homework.

- Systematically determine the number of primes between 1 and 100 using pattern recognition.

- Use the concepts of recognizing primes and patterns to solve problems.

- Use tables, pattern recognition, and solving a simpler problem to solve problems and make the results more general.

Morning

- Review homework. Have students swap with another classmate and compare answers.

- Give the students two numbers, one prime and one not. Have them compare different characteristics between them. Give them two other numbers of the same sort. Have them compare again. See if they can pull out the definition of a prime number without giving them the actual definition.

- Define prime and composite numbers. Have the students find all the prime numbers between 1 and 100. Discuss different strategies, including the Sieve of Eratosthenes. (See Wikipedia for the Sieve).

- Do 'Return of the Howling Dogs' on page 143. Have the students make a list to recognize the pattern.

- Split the students up into groups and give them each a number range greater than 100. (For example, 100-125.) Each group has to find a prime number in their range, and then switch with another group. The other group seeks to find a factor; if none can be found the number is verified as prime.

- Have students cut out 'trains' of length 1, 2, 3, 4, 5, and 6. Have them figure out how many ways a '6' train can be created using only '1' and '2' trains. For additional ideas, see the lesson plan on the NCTM website: [lesson plan](#)

Homework
- Have the students complete the following question, using a pattern. They must be able to show their pattern and how they got their answer. What is the digit in the ones place of 2^18?

Day: 9
Objectives
- Go over the homework, reviewing looking at patterns.

- Recognize patterns, diagrams, and models/simulations to determine ciphers.

- Write their own ciphers, then trade and solve one another's.

Morning
- Explore the Pig Pen Cipher (See Wikipedia for an example).

- Have students come up with their own sentences using the pig pen cipher, then trade with a partner to decipher them.

- Have students work on the 'Life on the Farm' problem on page 57.

Afternoon
- Have the students work on the different Fibonacci worksheets found at [Fibonacci worksheets](#)

- Allow students some time to work on their final projects while I meet with them individually to talk about the mid-session evaluation of student experience at CTY.

- John counted the lines of a page in his book. Counting by threes gave him a remainder of 2, counting by fives also gave him a remainder of 2, and counting by sevens gave him a remainder of 5. How many lines were on the page? Try the "Eliminate Possibilities" strategy!

Day: 10

Objectives

- Go over the homework. Have students swap with another classmate and compare answers.

- Introduce the idea of using tables and eliminating possibilities to solve problems.

Morning

- Tell students that I am thinking of a number between 1 and 100. Begin by having them randomly try to guess (for the sake of demonstration be sure they don't guess the number right away.) After a minute, stop guessing and discuss how we can organize their guesses or find the solution quicker. If an example is needed, ask "What questions could you ask me to narrow down the choices?" (e.g. Is it below 50? Is it prime?) Allow students to develop their own questions to eliminate possibilities and model the recording of them on the board. Discuss how eliminating possibilities helps to solve problems.

- Next, work through "Three Brothers Go A-Courtin' from pg. 51 Note the idea of listing or diagramming ALL possibilities and then eliminating from there.

- In pairs, have students solve and share "Penny's Dimes" on pg. 52. Have them present their answers and then discuss the results.

Afternoon

- Show the students how to do a Ken Ken ([Ken Ken website](#)). Have them work on Ken Ken puzzles for the majority of the afternoon. Allow them to time themselves, and compete against one another with the same puzzle. Teach them how to access the puzzles at home. Consider making a tournament, and offering harder and harder problems.

Homework

- Have the students do Problem 11 on page 71 of the text. Have them write at least five complete sentences.

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Day: 11
Objectives

- Review homework by sharing with a classmate.

- Use the strategy of Working Backwards to solve problems.

Morning

- Have each student swap homework and correct their classmates.

- Read through the introduction to the chapter (pages 329-332) about a vacation to Hawaii and working backwards.

- Have the students solve "Poor Choices" on page 332 in small groups.

- Have the students work through "Number Trick" on page 336 individually.

- Have students work though "A Rich Estate" on page 338, but simplify the numbers to be hundreds, not hundred thousands. Have them work in small groups.

Afternoon

- Have students work through 'Mints' on page 339 in the text. If they don't know about fractions we will have a mini lesson on what 1/3 and ¼ mean.

- Students will get to work on their final projects, which will be due on Friday.

Homework

- Have the students work through Problem Set A.4 in the text (page 347).

Day: 12
Objectives

- Review Homework.

- Review Using Patterns to solve problems.

- Students will use multiple strategies to solve problems.

- Review the homework. Have each student swap papers with a classmate and compare their answers.

- Tell students that I possess the power to read minds and demonstrate using power cards. Then, split the students up into small groups to examine the cards to try and find patterns. See if they can figure out why it works. For the full lesson, see Birthdays and the Binary System Lesson

Afternoon

- Read Math Curse by Jon Scieszka and Lane Smith and solve problems as we go through it.

Homework

- What is the sum o the first 100 whole numbers? Solve this problem in a manner that doesn't include adding 1 + 2 + 3 + 4... etc.

Day: 13

Objectives

- Review Homework

- Students will use a table and logic to problem solve.

- Review the problem solving strategies we learned about in the course.

Morning

- Teach the students what a matrix is. Ask them to list some of its qualities that would be good for problem solving, and some of its limitations.

- Have the students work in small groups to solve 'Favorite Sports' on page 79 in the text, using a matrix.

- Have students work on the logic puzzles found at logic puzzles We would use the computer lab if it were available.

- Have students write their own logic problems by first selecting a topic they want to write about (sports, movies, music, etc). Then have them create the answer- a matrix. *Then*, have them write the problem.

Afternoon

- Have students swap logic problems from the morning and do each other's problems.

- Review with the students the different methods we learned to problem solve. These include: Draw a diagram, make an organized list, find a pattern, eliminate possibilities, use logical reasoning and matrices, and make a table.

Homework
- Have each student write their own logic problem and bring it to class, with an answer on a separate sheet of paper.

Day: 14
Objectives
- Students will complete their Student Program Evaluations.

- Have students complete the post assessment.

- Finish Projects

Morning
- Student will swap their logic problems from the previous nights' homework and solve them.

- Students will complete their Student Program Evaluations.

- Students will begin their Final Assessment.

Afternoon
- Students will complete their Final Assessments.

- Students will work on their projects.

Homework
- Finish Projects!

Day: 15
Objectives
- Celebrate and share the new knowledge gained.

- Prepare for the open house

Morning

- Students will share their projects with their classmates.

Afternoon

- Students will prepare for the open house.

Homework

- It's the last day so… none.