

Inductive and Deductive Reasoning (INDE)

CTY Course Syllabus

Day 1- RECUSRIVE FORMULA AND INDUCTIVE REASONING

Objectives

- Review classroom rules to create a positive study environment
- Present the lesson plan for the summer session
- Introduce the first chapter of the course on “Recursive Formula and Inductive Reasoning”

Morning (165 min)

- Check if some of the students are already familiar with the content of the course.
- Start the course on “Recursive Formula and Inductive Reasoning”
 - For recursive formula,
 - Give several examples of sequence of numbers with a clear pattern of recursion and make the students complete the list;
 - Also give example where no clear pattern appears;
 - Solicit different examples from students;
 - State the definition of a recursive formula
 - For inductive reasoning,
 - Give few picture examples involving induction patterns (use geometric form or numerical numbers); let student find the pattern; solicit other examples from them
 - State the definition of an inductive reasoning
 - Compare the inductive reasoning with deductive reasoning
 - Give example involving both inductive and deductive reasoning and let students find which one is inductive and which one is deductive

Afternoon (50 min.)

- Review of the definition of recursive formula and inductive reasoning
- Let students work on their own examples of the two concepts and let some of them present their finding on the board and create a dialogue between the student on the board and the class

Late Afternoon (45 min)

- Have students read Chap. I of the Number Devil book;
- Do Day 1 lesson summary
- Give homework assignments to work on when they will be outside the classroom

Day 2- EXPLICIT FORMULA AND FIGURATE NUMBERS

Objective

- Define explicit formula and distinguish between explicit and recursive formula
- Apply explicit and recursive formula to figurate numbers

Morning (165)

- Day 1 Lesson Review
 - Review the main content of Day 1 lesson on recursive formula and inductive reasoning
 - Revisit some of the examples provided during Day 1 lesson
 - Ask students if they have any questions, difficulties, remarks, or comments on Day 1 lesson
- Introduction to Explicit Formula Lecture
 - Have students perform traffic jam and toothpick staircases activities
 - Have them find explicit formula for these two activities
 - Have them find explicit formula from some Day 1 lesson activities
 - Provide the definition of explicit formula
 - Examples of Explicit Formula
 - Do a hand-on-activity to provide an example of explicit formula and have students give few examples of explicit formula that they might be aware of.
 - Introduce factorials as an example of explicit formula
 - Have the students work independently on some example of factorials
- Have students figure out the difference between recursive and explicit formula
- Lecture on Introduction to Figurate Numbers
 - Have students perform a hand-on-activity on triangular numbers
 - Have students figure out the pattern in the triangular numbers
 - Define the concept of a Figurate Number and give few examples of such number
 - Introduce Gauss's formula using a step by step approach
 - Have students give examples of Figurate Numbers they might know, and if they finish early have them work on Square and Pentagonal Numbers.
- Summary of the lesson

Afternoon (50 min)

- Have a student read Chap 2 of the Number Devil book and explain the concepts
- Student write a summary of the chapter
- Split students into several teams and have each team work on cube manipulatives.

Late Afternoon (45)

- Let students work on the topic assignments (which I will give) and turn in their work
- If some students finish early, make them work on finding explicit formula for other Figurate Numbers than Triangular, Square, Pentagonal Numbers they might know.

Day 3 - DEDUCTIVE REASONING

Objective

- Introduce the notion of deductive reasoning and its distinction from inductive reasoning
- Distinguish truth from validity in simple deductive arguments
- Recognize and determine validity of syllogisms

Morning (165 min)

- Turn back graded assignment of Day 2 to students
- Point out remarks where there were student common mistakes in the students' homework
- Revisit Day 2 lesson on explicit formula and figurate numbers.
- Ask a thought questions

- Can animal use inductive reasoning?
- Have students write down and present independent answers
- Discuss Pavlov's dog experience, Beeline, instinctive vs. inductive reasoning
- Deductive Reasoning Lecture
 - Give few examples of deductive reasoning.
 - Defining the concept of deductive reasoning
 - Have students give few examples and explain their choice
- Summary of the lesson

Afternoon

- Read Chap. III of the number devil book
- Student write a summary of the chapter

Check student background on prime numbers

Have then construct the Sieve of Eratosthenes (through 100)

Discuss different mean to check whether a number is prime number

Discuss conjectures: Goldbach and Twin Prime conjecture etc.

Present few computational tools to define prime numbers

Late Afternoon

- Students keep working on their own figurate numbers
- Have write down what learn from today's lesson.
- Collect the notebooks for review.

Day 4 – DEDUCTIVE REASONING

Objective

- Introduce the concept of deductive reasoning and its distinction from inductive reasoning
- Distinguish truth from validity in simple deductive arguments
- Recognize and determine validity of syllogisms

Morning (165 min)

- Give back students notebook.
- Revisit Day 3 lesson on deductive reasoning
- Have students practice few syllogism statements for Day 3 lesson by presenting what they did for their homework. Their work should include truth and validity of a syllogism.
- Practice the use Venn diagram to show the validity of a deductive reasoning statements.
- Discuss some practical applications of primes numbers.
- Contrapositive Lecture
 - Introduce contrapositive
 - Have students write/present few contrapositive
- Distinguish between Inductive and Deductive Reasoning
 - Present few examples of deductive and inductive reasonings

- Have the student write up why each of these examples correspond to deductive or inductive reasoning
- Discuss and debate the answers

Afternoon (50 min)

- Read Chap IV of the number devil.
- Student write a summary of the chapter
- Then
 - Split the students into 3 pairs.
 - First group discuss square root of a positive number, the other one decimal number, and the third group exponents.
 - Discuss the irrational nature of square root of 2.

Late Afternoon (45)

- Make students continue working on their own figurate number write ups
- Have them write a summary of what they learn today
- Give homework assignments on Day 4 lessons
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Day 5 – TRUTH, VALIDITY, AND SOUNDNESS

Objective

- Master the use of premises to reach conclusions and analyze the validity of conclusions
- Develop the ability to determine whether syllogisms are valid or invalid

Morning (165 min)

- Turned back graded assignment for Day 3+4 lesson to students
 - Point out remarks common mistakes in the students' work and propose how to fix them
 - Talk about good mathematic writing
- Revisit Day 3+4 lessons on deductive reasoning
 - Have students practice few syllogism statements for Day 3+4 lesson
- Lecture on Introduction to using premises to reach valid conclusion
 - Define premise statements and give some examples of premises chosen from
 - Geometry,
 - English language
 - Check if the students are already familiar with some premises
 - Have them repeat these premises to class
 - Let the class check whether they are accurate or not
 - Emphasize distinction between truth and validity
- Present the Chart Height Problem
 - Let students work in teams to solve the problem

Afternoon (50)

- Read Chap V of the number devil.
- Demonstrate the “a square number” is the sum of two triangles statement
- Present a thought question

- Ask students if they know what an axiom is?
- What axiom do they use in everyday life?
- Create dialogue between students about what their daily axiom is?

Late Afternoon (45)

- Discuss using counterexample for some mathematical statement
- One-on-one meeting with students to discuss write-ups quality
- Have students write in a few sentences what they have learned from today's lesson.

Day 6 - PROOFS

Objective

- Explain simple graphical and algebraic proofs

Morning (165 min)

- Revisit Day 5 lesson contents
- Give students a list of 5 elements and let them draw a Venn Diagram
- Make students write valid and invalid syllogism
- Lecture on Introduction to Proofs
 - Prove distributive property
 - Give few examples to illustrate the property
- Provide proof of $0.999\dots = 1$
- Provide proof of the diagonal length of the unit square
- Triangle and trapezoidal area formula
 - Check student background on some special geometrical surface areas formula
 - Make students prove the triangle and trapezoid area formula on their own

Afternoon (50)

- Read Chap VI of the Number Devil Book
- Student write a summary of the chapter
- Make students continue the proof of the formula of the area of trapezoid
- Proof of the Pythagoras theorem
 - Using the curriculum binder
 - Using diagrams
- Make students prove the area of a circle using construction paper and scissors

Late Afternoon (45)

- Make student work individually or in pairs on graphical proof assignments
- Have them write up what they learned from today's lesson.

Day 7 – LIMIT AND FIBONACCI NUMBERS

Objective

- Introduce students with the notion of limit and use of relevant notation
- Explore Fibonacci sequence, golden ratio, and a few applications

Morning (165 min)

- Revisit Day 6 lesson on graphical and algebraical proof.
 - Give few example of proof (even + even = even)
 - Have students try to prove the other 2 combinations on their own.
 - Review the fundamental theorem of arithmetic
 - Define rational and irrational numbers.
- Lecture on Fibonacci Numbers and the Golden Ratio (25 min)
 - Define and have students compute the first 17
 - Have students make a ratio sequence and find limit
- Lecture on Intro to Limits (20 min)
 - Give “Definition”, walk through several examples, introduce proper notation, point out the connection with inductive reasoning
 - Have students compute a few infinite limits on their own

Afternoon (50 min)

- Read and explain the concept in Chap VII of the Number Devil book.
- Student write a summary of the chapter
- Continue limits
 - See if students can find the infinite limit trick
 - Explain why this trick works Worm Problems (students work in groups)

Late Afternoon (45 min)

- Continue limit
- Have student write what they have learn from today’s lesson.

Day 8 – MODULAR ARITHMETIC

Objective

- Add and subtract in modular arithmetic
- Prove of divisibility by 3 and 11
- Pascal’s triangle patterns

Morning (165 min)

- Revisit Day 7 lesson.
 - Finish discussion of limits
 - Do limits where n approaches a fixed number
 - Discuss the definition of division and $0/0$
 - Have half of the class do right-sided limit and half do left-sided limit
 - See if students notice another explicit formula $(n+4)$ and prove using difference of 2 squares
- Divisibility by 3
 - Finish discussion of limits
 - Lecture on modular arithmetic
 - Students complete worksheet individually on modular arithmetic

- Instructor guides students through the proof of divisibility by 3

Afternoon (50)

- Read and explain the concept in Chap 8 of the Number Devil
- Student write a summary of the chapter
- Introduce Pascal's Triangle
 - Notice patterns
 - Color in even/odd numbers

Late Afternoon (45)

- Keep working on Pascal Triangle
 - Find different pattern in the triangle
 - Tied the PT with combinatory
- Mid-course self-evaluation
 - Students write about the performance in certain areas of the course

Day 9 – SYMBOLIC LOGIC

Objective

- Construct truth table
- Translate symbolic logic into English.
- Pascal's triangle patterns

Morning (165 min)

- Revisit Day 8 lesson.
- Introduction to Symbolic Logic
 - Students follow along with fill-in notes
 - Concepts: not, and, or, if-then, if-and-only-if, truth tables
- Painted Cube project
 - Major course project
 - Students work individually
 - Formal Write-up is required

Afternoon (50)

- Continue working on practicing Symbolic Logic
- Continue work on Painted Cube project

Late Afternoon (45)

- Read and explain the concepts in The Number Devil Chap 9
- Student write a summary of the chapter
- Have student write a summary of what they learned in today's lesson.

Day 10 – SYMBOLIC LOGIC, CONT'D

Objective

- Construct truth table
- Translate symbolic logic into English.
- Pascal's triangle patterns

Morning (165)

- Seating Puzzle
 - Students will use a set of premises to figure out where to sit.
- Continue the work on Painted Cube Project

Afternoon (50)

- Read and explain Chap X of the Number Devil book
- Finish the Painted Cube Project

Late afternoon (45)

Fractal Video/Talk

- Students watch a video on fractals and their applications
- Instructor gives lecture on Fractal Dimension

Students write what they have learn from the day's lesson.

Day 11- LIMIT AND SYMBOLIC LOGIC

Objective

- Construct truth table
- Translate symbolic logic into English.

Morning (165 min)

Carpool Logic Problems

- Students work individually evaluating truth statements about the axioms
- Students present solutions afterwards
 - Every student writes his/her model on board
- Define Sigma Notation and give a few examples
- Define an infinite sum as the limit of partial sums
- Use "walking" and "pie" examples to illustrate the concept of an infinite sum

Lecture

Koch's Snowflake Fractal

- Students create their own snowflake on grid paper
- Relate area and perimeter to limits

Afternoon (50 min)

Number Devil Chapter 11

- Instructor/TA read and explain concepts

Finish Koch's Snowflake Fractal

- Explain Perimeter/Area Paradox
- Talk about the perimeter and area of Sierpinski's Triangle
- Talk about Coastlines

Late Afternoon (45 min)

Coastline of Latin America

- Discuss distance of coastlines and relate to fractals

Truth Table Practice

- Instructor presents example
- Students try some individually

Students write what they learn from the day's lesson.

Day 12 - PARADOXES

Objective

- Articulate conflicting lines of reasoning that emerge from paradoxes and assess these lines of reasoning.

Morning (165 min)

Student Program Evaluations

Translating Symbolic Logic Statements (30 min)

- Go over first 2 pages orally
- Students work on next 3 pages
- Go over packet as a class

Analyzing Paradoxes (35 min)

- Define paradox, give some simple examples
- Instructor walks students through Newcomb's Paradox
- Students answer questions in pairs Continue Paradoxes (40 min)
- Finish Newcomb's Paradox

- Present the Barber Paradox

Afternoon (50 min)

Number Devil Chapter 12

- Instructor/TA read and explain the content of the chapter
- Students write a summary of the chapter

Triangle Area Paradox on graph paper

- Students draw shapes on graph paper to try to find the error

Late Afternoon (45 min)

Finish Discussion of Paradoxes (45 min)

- Hangman's Paradox
- Russell's Paradox

Daily lesson summary write-up.

Day 13 - INFINITY

Objective

- Distinguish between countable and uncountable infinities
- Experience the seemingly paradoxical results of attempting to calculate with infinity.
- Find explicit formula and recursive formula for the tour of Hanoi problem.

Morning (165 min)

Introduction to Proof by Mathematical Induction

- Define proof by mathematical induction
- Application to several examples

Infinity Lecture

- Define countable and uncountable infinity
- Present/solicit examples
- Present Cantor's diagonalization argument

Afternoon (50 min.)

Post-Assessment Test

Late Afternoon (45)

Tower of Hanoi Activity

- Students work individually or in pairs

- If finished, they can work on “The Hardest Logic Puzzles of All Time”

Daily write up of students have learned.

Day 14 – REASONING OUTSIDE THE BOX AND CLOSURE

Objective

- Practice creative solving problem skills.

Morning (165 min)

Lateral Thinking Puzzles

Finish Mathematical proof by Induction examples left as homework

Tower of Hanoi Activity, Cont'd

- Students work individually or in pairs

Thought Question

Is math discovered or created?

Students write about the question, then each student share, then open discussion.

Student write short letter to instructor and TA.