

# CTY Course Syllabus

## Great Discoveries in Mathematics

### **Day 1: What is Math?**

Towers of Hanoi  
James' Puzzle and Rev's Puzzle  
Pre Assessment  
Introductions  
Reading: *On Problems*  
Mathematical Problem Solving Techniques  
Rules and Honor Code  
108: A Demonstration on Asking Mathematical Questions  
The Rama Nujan Stone

### **Day 2: Geometry**

Archimedes Flower  
Euclid's Elements  
The Axiomatic Method  
Geometric Constructions  
Archimedes and Crowns  
Gauss and Non-Euclidean Geometry  
Hyperbolic Manifolds

### **Day 3: Geometry**

Pythagorean Triples  
The Many Proofs of the Pythagorean Theorem  
The Story of Pi  
Measuring Angles with Radians  
Pi vs. Tau

### **Day 4: Number Theory**

Consecutive Sums  
Polygonal Numbers  
Reading: *A Mathematicians Lament*  
Discussion: What is Mathematics?  
Prime Numbers  
Perfect Numbers  
Marin Mersenne  
GIMPS and Large Primes  
Fermat's Last Theorem  
Fermat Numbers  
Large Primes  
Sieve of Eratosthenes

### **Day 5: Different Number Systems**

Last Digit of the Largest Prime  
Cubes of Two Digit Numbers  
Roman Numeral  
Egyptian Numerals  
Babylonian Numerals  
Mayan Numerals  
The Euclidean Algorithm  
Discussion: Is Mathematics Invented or Discovered

**Day 6: Game Theory**

The Almost and Over-Eager Problems  
The Game of 20  
The Game of Nim  
Sprouts  
Philosophers Phutball  
Rigged Tac Toe

**Day 7: Graph Theory**

The Divisor Game  
Sieve of Great Discoveries  
The Seven Bridges of Königsberg  
Leonhard Euler  
The Graph Coloring Game  
Party Problem  
Directed Graphs and Winning Strategies

**Day 8: Analytic Geometry and Algebra**

Cryptography  
Modular Arithmetic  
Cardano and the Cubic  
Polynomials and Solvability  
Evariste Galois  
Hexaflexagons

**Day 9: Sequences, and Series**

Fractions to Decimals and Back  
Sequences  
Fibonacci Sequence  
Zeno's Paradox  
Geometric Series  
Infinite Series  
Sieve of Great Discoveries Work

**Day 10: Calculus and Physics**

Koch's Snowflake  
Liebniz and Inverse Triangle Numbers  
Galileo and Falling Bodies  
Newton and Calculus  
Derivatives and Tangent Lines  
Integrals and Areas

**Day 11: Sets**

Sets  
Barber's Paradox  
Equinumerous and Cardinality  
Sieve of Great Discoveries Work  
Einstein's Puzzle  
Two Sundry Problems

**Day 12: Infinity**

Hilbert's Hotel  
Georg Cantor  
The Naturals and Countable Infinity  
The Reals and Uncountable Infinity  
Transcendental Numbers

**Day 13: Paradoxes**

Missing Square Paradox

A Potato Paradox

Speed Paradox

The Berry Paradox

$1=2$  Paradox

Barber's Paradox

The Birthday Paradox

Banach-Tarski

Newcomb's Paradox

**Day 14: Presentations and 20<sup>th</sup> Century Mathematicians**

Post Test

Sieve of Great Discoveries Write-up

**Day 15: Wrap Up and Goodbye**

Sieve of Great Discoveries Handout

Goodbyes