

Probability and Game Theory – Course Syllabus

DATE	ACTIVITY	CONCEPT
Sunday	Learn names; introduction to course, introduce the Battle of the Bismarck Sea as a 2-person zero-sum game.	
Monday Day 1	<p>Morning: Pre-test for assessment.</p> <p>Lecture: Review of Set Theory (Instructor - I)</p> <p>Lecture: Enumeration of sets (I) (Challenge Problem #1: generalize addition principle to the property of inclusion/exclusion)</p> <p>Afternoon:</p> <p>Lecture: Permutations and Combinations – including many examples (I & TA)</p> <p>(Challenge Problem #2: Soldiers in a field) (Challenge problems #4,5,6 on partitions of integers)</p> <p>Evening: Work quietly on first 2 written assignments. (I & TA)</p>	<p>-Sets, elements, subsets, universal sets, empty sets.</p> <p>-Set operations: union, intersection, complements, disjoint sets.</p> <p>- Properties of these operations: commutative, associative, distributive laws; DeMorgan's Laws. Venn diagrams</p> <p>- Addition principle.</p> <p>- Multiplication principle, Cartesian product of sets, power sets</p> <p>- Enumeration when order of selection matters and when it does not. Selection with and without replacement.</p> <p>- multinomial coefficients</p> <p>- Recursion/Induction</p> <p>- partitions of integers</p> <p>- Sets and enumeration.</p>

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Tuesday Day 2	<p>Morning: Lecture: binomial theorem and set partitions (I)</p> <p>(Challenge problem #3: generalize the binomial theorem to a multinomial theorem.)</p> <p>Lecture: Introduction to probability theory; Uniform probability models (I)</p> <p>Challenge problems: Lemon Candy Problem, Putnam Problem, Buffon's Needle.</p> <p>Afternoon: Lecture: Birthday problem (TA)</p> <p>Lecture: Expected values/examples (I)</p> <p>Evening: Return and go over assignments 1 & 2. Work quietly on assignments 3 & 4. (I)</p>	<ul style="list-style-type: none"> - Binomial coefficients; Pascal's triangle and properties. - Multinomial coefficients and enumeration of set partitions - Experiments, outcomes, sample spaces, events - Probability Models, probability of a union of events - Uniform sample spaces, using enumeration to compute probabilities. - Infinite Sample Spaces - Probability of a complement - Expected values; Independent Events - Binomial theorem, probability.

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<p>Wednesday Day 3</p>	<p>Morning: Quiz I on sets and enumeration</p> <p>Work quietly on assignments.</p> <p>(Challenge problems #7-9: enumeration of compositions; challenge problem on Ramsey theory, Mr. Spock logic problem)</p> <p>Lecture: Matrix algebra (I)</p> <p>Return and go over Quiz I</p> <p>Afternoon: Class Discussion: Introduction to game theory – the resolution of the Bismarck Sea Battle (I)</p> <p>Video – “Zero-sum Games” from the ‘For All Practical Purposes’ series of educational mathematics videos.</p> <p>Evening: Work quietly on assignment #5 (TA)</p>	<p>- Ordered partitions (compositions)</p> <p>- Matrices, scaling, addition of matrices, dot product of vectors, Matrix multiplication. - Matrix determinants and inversion (2x2 case only), solving systems of linear equations.</p> <p>- Game trees, game matrices, dominant strategies, minimax techniques and saddle points, higher order dominance.</p> <p>- Expected values and matrix algebra.</p>

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Thursday Day 4	<p>Morning: Work quietly on assignment #5 (TA)</p> <p>Class Discussion: Strictly Determined Games, Continued. (I)</p> <p>Lecture: Introduction to Non-strictly Determined Games (I)</p> <p>Challenge problem – “Grid” Problem</p> <p>Afternoon: Lecture: Introduction to Linear Programming.</p> <p>Video – “Linear Programming” from the ‘For All Practical Purposes’ series of Educational Videos.</p> <p>Evening: Work quietly on assignments #5 & 6 Read Chapters 1-2 of text. (TA)</p>	<ul style="list-style-type: none"> - More on dominance, saddle points and minimax techniques. -movement diagrams. - Value of a game, fair games, translating a game. Saddle points are equivalent and interchangeable. - Introduction to non-strictly determined games. Repeated play and mixed strategies. - Mixed strategies, probability vectors as strategies - Expected payoffs as matrix multiplication. - Expected Value Principle (when the opponent’s strategy is known) - method of equalizing expectation (when the opponent’s strategy is not known) – 2x2 case only; - Probabilities & binomial coefficients - Graphical Solutions, feasible points and optimal points, corner point theorem, marginal values. - Solving Strictly Determined Games, Dominance

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Friday Day 5	<p>Morning:</p> <p>Review assignments 4 & 5 (TA & I)</p> <p>Quiz II on binomial theorem, probability and expected value.</p> <p>Lecture: Non-strictly Determined Games, Continued (I)</p> <p>Afternoon:</p> <p>Lecture: More on Linear Programming: marginal values and GLP software demos (I)</p> <p>Class Activity: Enact the Grid problem</p>	<ul style="list-style-type: none"> - Equalizing expectation in $2 \times n$ case or the $m \times 2$ case. Inactive strategies. - Equalizing expectation in the 3×3 case; Williams' theorem that for any game, the solution is the same as that of some square subgame. - Williams' method of oddments (2×2 case) - Sensitivity Analysis; Standard Form problems - Monte Carlo Techniques in Probability Theory
Sunday	<p>Evening:</p> <p>Work quietly on assignments #6 - #7 (I)</p> <p>Read Chapter 3 of text</p>	<ul style="list-style-type: none"> - Zero-sum games; strictly determined and non-strictly determined games.

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Monday Day 6	<p>Morning: Work quietly on assignments #6 - #7</p> <p>Lecture: Duality in Linear Programming and its economic interpretation. (I)</p> <p>Lecture: Introduction to the Simplex Method. Sensitivity Analysis (I)</p> <p>Class Discussion: Buffon's Needle Problem: how to set it up (I)</p> <p>Afternoon:</p> <p>Lecture: Simplex Method continued. (I)</p> <p>Return and go over Quiz II (I)</p> <p>Class Discussion of the Buffon's Needle Challenge Problem (I)</p> <p>Brief introduction to variable sum games (I)</p> <p>Evening: Go over assignment 5 Work quietly on assignments #7 - #8 (I)</p>	<ul style="list-style-type: none"> - Marginal values as decision variables - Linear Programming via simplex method. - Sensitivity Analysis - Infinite Sample Spaces - Rules for pivoting - Introduction to trigonometric functions - ordered pair payoffs, movement diagrams, Nash equilibria - Solving linear programming problems algebraically.

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<p>Tuesday Day 7</p>	<p>Morning: Work quietly on assignments #7 - #8 (TA)</p> <p>Movie: A Beautiful Mind</p> <p>Afternoon:</p> <p>Class Discussion: The Jamaican Fishing Problem (TA)</p> <p>Make-up Quiz II</p> <p>Evening: Work quietly on assignments #8 & #9 (TA) Read Chapter 4</p>	<ul style="list-style-type: none"> - The life of John Nash - Application of game theory to Anthropology - Games against Nature - Probability and expected values - Tree diagrams; information sets, games of partial information, backwards induction.
<p>Wednesday Day 8</p>	<p>Morning: Return make-up quizzes</p> <p>Lecture: Linear Programming: Solving minimization problems via duality. (I)</p> <p>Lecture/Demo: Using Software to solve LP problems. (I)</p> <p>Class Discussion: Guerilla Warfare & Missile Games (TA)</p> <p>Afternoon:</p> <p>Lecture: Linear Programming applied to game theory (I)</p> <p>Evening: Work quietly on assignments #8 - #10 (TA)</p>	<ul style="list-style-type: none"> - Duality- solving dual problems via the simplex method - Working with Mathematica - Using Expected Values as payoffs and partitions as strategies. - Solving $m \times n$ games via simplex method - Proof of the minimax theorem

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<p>Thursday Day 9</p>	<p>Morning:</p> <p>Work quietly on assignments #8 - #10 (TA)</p> <p>Return and go over assignment #6; review of solving games via linear programming (I)</p> <p>Lecture: Games in Extensive Form (I)</p> <p>Lecture/Class Demo: Using Excel to solve LP problems and games. (I)</p> <p>Afternoon:</p> <p>Return & go over assignment #7</p> <p>Quiz III on Zero-sum Games</p> <p>Evening:</p> <p>Assignments #9 & #10 (I) Read Chapter 7</p>	<p>-Game Trees, Information sets; Extensive and Normal Forms are equivalent; introduce Cuban Missile Crisis</p> <p>-Solving zero-sum games via linear programming</p> <p>- Why oddments works; why equalizing expectations does not work for strictly determined games; games with more than one optimal solution.</p>

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Friday Day 10	<p>Morning:</p> <p>Return and go over Quiz III</p> <p>Lecture: Applications to Business. (I)</p> <p>Lecture: Games Against Nature (TA)</p> <p>Afternoon:</p> <p>Lecture: Variable Sum Games (I)</p> <p>Video: "Prisoner's Dilemma" from the 'For All Practical Purposes' series of educational mathematics videos. (I)</p>	<ul style="list-style-type: none"> - Games of partial information - Playing against non-rational opponents - Axioms for playing games against nature - Milnor's axioms - Dominance, Nash equilibria - Payoff polygons, Pareto Optimality - Equalizing and prudential strategies - Iterated Games; 'Tit for Tat - Prisoner's dilemma; chicken
Sunday	<p>Evening:</p> <p>Work on Assignments #9 - #10</p> <p>Read Chapters 11 & 12 (I)</p>	<ul style="list-style-type: none"> - Games Against Nature - Finding Nash Equilibria, SSS games.

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Monday Day 11	<p>Morning:</p> <p>Lecture: Variable Sum Games, Continued (I)</p> <p>Lecture: Strategic Moves (threats and promises) (I)</p> <p>Class tournament – Wholesalers vs. Retailers. (I & TA)</p> <p>Lecture: 2x2 Ordinal Games (I)</p> <p>Afternoon:</p> <p>Lecture: Ordinal games and the theory of moves. Applications to the Yom Kippur War. (I)</p> <p>Evening: Assignments #11 & #12 Read Chapters 12 & 14 make-up quiz III (TA)</p>	<ul style="list-style-type: none"> - Games solvable in the strict sense - Communication between players in variable sum games - Games of partial information; communication via actions. - Applications – the superpowers arms race, Cuban missile crisis. - Nash equilibria, dominance, Prisoner’s dilemma - Sequential games; non-myopic equilibria - Relaxing assumptions of game play: communication, sequential play, initial states - Strategic moves, applications of the prisoners dilemma
Tuesday Day 12	<p>Morning:</p> <p>Lecture: Theory of moves, cont. (I & TA)</p> <p>Movie: 13 Days</p> <p>Afternoon:</p> <p>Class discussion on Evolutionary Stable Strategies (I)</p> <p>Evening: Assignments #11 & #12 Read Chapters 14 & 15 (TA)</p>	<ul style="list-style-type: none"> - The Cuban missile crisis - Applications of game theory to biology - Evolutionary Stable Strategies - Ordinal Games, Theory of moves

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<p>Wednesday Day 13</p>	<p>Morning: Work quietly on homework (TA) Go over homework (I) Class Discussion: n-person games (I) Afternoon: Lecture: n-person games, continued (I) Evening: Catch up on all assignments Read Chapters 19-22 (TA)</p>	<ul style="list-style-type: none"> - Prisoner's dilemma in Puccini's opera "Tosca", battle of the sexes, etc. - n-person version of Prisoner's dilemma; Tragedy of the Commons - Coalitions, security levels, n-person prisoners dilemma; - Games in characteristic function form - n person Prisoners' Dilemma - Applications to the football draft
<p>Thursday Day 14</p>	<p>Morning: - Jeopardy-type class tournament (TA) Study for final Exam (I & TA) Afternoon: Final Exam Evening: PBS video: A Brilliant Madness Fill out SPEs (I)</p>	<ul style="list-style-type: none"> - review all topics -The life and work of John Nash
<p>Friday Day 15</p>	<p>Morning: Return Exams Hand out Solutions to Challenge Problems Class Party (bring games) Afternoon: Closing Ceremony</p>	