

WEEK ONE: Introduction to Concepts in Game Theory and Probability and Zero-Sum Games

<u>Day</u>	<u>Session</u>	<u>Activities</u>
00	Evening	<ol style="list-style-type: none"> 1. (20mins) <u>Welcome and Introductions</u>: Brian and Andy introduce themselves, their backgrounds, their interests. Students play “Who Is It?” game. Each student receives a notecard and writes 3 facts about themselves on it. Groups are broken up into two teams, and one card is read off at a time. Groups must guess who on the other team the card belongs to, winning points. 2. (10mins) <u>Prior Knowledge</u>: With intros out of the way, what do people know about Game Theory and Probability? What is each? What comes to mind for each one? Give students three minutes to think, then let students write answers up on board and discuss. 3. (30mins) <u>Syllabus Overview</u>: Give full definition of Game Theory and Probability. Walk through syllabus with students, explain goals of the course and overall arch of the class. Discuss and model norms for behavior in the course. Introduce “Bidding” and points
01	Morning	<ol style="list-style-type: none"> 4. (45mins) <u>Pre-Test</u>: Re-explain purpose of pre-test and emphasize diagnostic nature rather than evaluative. “You are supposed to do terribly.” Have students read Game Theory article after while others are finishing. Have students theorize RPS strategies. 5. (40mins) <u>Introduction to Basic Terms</u>: What’s in a game? Model; Game; Players; Strategy; Outcome; Payoff; Rationality; Perfect Information; Beliefs. Write definitions out alongside description of “Our First Game,” Rock Paper Scissors (no matrix, just describe). Have students try to write out the specifications of a different game/situation using the terms properly 6. (30mins) <u>Modeling Practice</u>: Students continue work on their models and discussion of how various models might work. Discussion of trying to simplify and hyper-specify in order to properly predict any scenario, and have students re-model based on those specifications 7. (30mins) <u>Ultimatum Game and Discussion</u> 8. (15mins) <u>Stretch Break</u>: Establish norms for stretch breaks (go to bathroom, leave together, gather on time) and go on break 9. (15mins) <u>Rock Paper Scissors Tournament, Pt. 1</u>: Students will play their first tournament of Rock, Paper, Scissors. Have students record their exact game history (wins/losses) through best of three. Wrap up morning session for Lunch.

Day	Session	Activities
	Afternoon	<ol style="list-style-type: none"> (45mins) <u>Introduction to Probability</u>: Switch gears with students into Probability. Introduce basic notation, simplification, Sample Spaces, and problem solving. Theoretical/Experimental probability, Conditional Probability (15mins) <u>RPS Tournament Analysis</u>: Give them time to calculate out the probability that they would have won the RPS tournament. Review process. (30mins) <u>Yahtzee</u>: Introduce rules, create groups, and have them play 3 rounds. Winners get points. (30mins) <u>Yahtzee Probability Analysis</u>: Have students attempt to calculate out the probabilities of each combination in Yahtzee.
	Evening	<ol style="list-style-type: none"> (05mins) <u>Welcome</u>: Set norms for evening sessions: TA-led, mostly review of day and solidification of concepts (30mins) <u>Probability Exercises</u>: Students practice probability exercises and review (45mins) <u>Uses of Game Theory</u>: Students read and take notes on Dixit/Nalebuff Chapter 1 (10mins) <u>Stretch Break</u> (20mins) <u>Discussion</u>: Strategic Thinking and applications from Dixit/Nalebuff. Other examples of “Leading from Behind” and so on. (05mins) <u>Exit Ticket</u>: 0-10 on Pacing, Difficulty, Comfort, Excitement
<u>02</u>	Morning	<ol style="list-style-type: none"> (05mins) <u>Daily Rundown + Where we were/Where we are/Where we’re going</u> (45mins) <u>What’s the Difference?</u>: Have students compare their understanding of two different situations and see what the major difference is in their payoff/outcome structures. A football game, versus a big band’s concert. Segue into definition of Zero-Sum versus Non-Zero-Sum, and other specifications of our most simplified models (2-person, 2-strategy, simultaneous, perfect info) (30mins) <u>Basic Game Models</u>: Revisit Rock Paper Scissors again and model out using “Normal Form” matrix. Be sure to describe the payoff notation. Have students create matrices for their own games from yesterday’s exercises (15mins) <u>Stretch Break</u> (20mins) <u>A Fascinating Game</u>: Introduce “A Fascinating Game” and have students reflect on outcomes (60mins) <u>Equilibria</u>: Work through solutions to “A Fascinating Game” and finding the “value” of the game. Definitions for Saddle Points, Dominance, Equilibria, and Value. Maximin/Minimax. Have students find values for their own matrices, if any. Open discussion of different insights. Re-emphasize rationality definition. (30mins) <u>Lotteries and Expected Value</u>: Tell students we’re doing a lottery, first without an entry fee, and then with an entry fee. Play for points. See if students can figure out what their probability is of winning, then ask how they strategize as to whether to play or not (voluntary entry versus mandatory entry, too).
	Afternoon	<ol style="list-style-type: none"> (30mins) <u>Matching Pennies</u>: Have students play and analyze “Matching Pennies” game. Get student input on their strategies and why - what is optimal? Can we solve this using what we know now? (20mins) <u>Expected Value</u>: Segue into basic calculation of expected value. Start with bets on flipping a coin, move to rock paper scissors again. Return to Lottery and how to calculate when to get in and when to get out (60mins) <u>Mixed Strategies</u>: Apply expected values to our expected payoffs in games and mixed strategies for 2x2 games.

Day	Session	Activities
	Evening	<ol style="list-style-type: none"> 1. (15mins) <u>Prescriptive vs. Descriptive</u>: Describe the difference between descriptive and prescriptive analysis - where Game Theory falls in this spectrum and how Equilibria move us towards prescriptive. 2. (45mins) <u>Mixed Strategies, Continued</u>: Solidify mathematical basis of mixed strategies and reveal Oddments method to students for expedient solving. Challenge students to prove its validity. 3. (30mins) <u>Matrix Exercises</u>: Worksheet of exercises, review, solidify understanding 4. (30mins) <u>“Beating Vegas:”</u> Application of expected value into real world scenarios article
03	Morning	<ol style="list-style-type: none"> 1. (05mins) <u>Daily Rundown + Where we were/Where we are/Where we’re going</u> 2. (20mins) <u>Basic Game Modeling Review</u>: Review what sort of games we can now solve and how to do each step. Clarify and solidify mixed strategies/oddments foundations. 3. (20mins) <u>Matrix Exercise Review</u>: Review matrix exercises from previous night to verify understanding 4. (20mins) <u>Intermediate Game Modeling</u>: Discuss the basic parameters of larger games and exactly how they differ from smaller games 5. (75mins) <u>Solving Games - Equilibriums, Dominance, Saddle Points, Mixed Strategies</u>: Work with students to solve a handful of larger games ($n \times n$) using the same methods we had used for smaller games and explain the basic logic behind how they are still applicable. Review saddle points and movement diagrams especially, given that we glossed over them for 2×2 games. Run through exercises as a class and individually. 6. (45mins) <u>Higher Order Rationality</u>: Explain new definitions for higher order rationality concepts and play the “Islander” game with students to see if they can solve. Discuss applicability and solution.
	Afternoon	<ol style="list-style-type: none"> 1. (30mins) <u>Modeling Practice</u>: Have students attempt to create and model situations that involve more than just two strategies per player. Discuss limitations, applicability, etc. 2. (60mins) <u>2 x N and M x 2 games</u>: Move towards the method of graphical solutions for $2 \times n$ and $m \times 2$ games. Run through both methods slowly, given the difficulty of concepts 3. (30mins) <u>Jamaican Fishing</u>: Explain the Jamaican Fishing application and lead discussion around the general applicability of “solved” games. Introduce the idea of a game against nature; tease concept for later.

Day	Session	Activities
	Evening	<ol style="list-style-type: none"> <li data-bbox="436 196 1919 261">1. (30mins) <u>Reading: 1-26 in Straffin</u>: Students read through Straffin’s explanations of the day’s material and solidify their understanding with further explanations. <li data-bbox="436 269 1934 334">2. (45mins) <u>Graphical Solutions Practice + Review</u>: Provide exercises and walk through a single example together with the class again to ensure students are solving the method correctly (and not in reverse) <li data-bbox="436 342 1923 407">3. (45mins) <u>Reading: 56-84 Dixit/Nalebuff and Discussion</u>: Discuss the morality of dominant strategies, zero-sum games, and generally seeking the highest payoffs. Run through applications of Dixit’s examples to other situations.
04	Morning	<ol style="list-style-type: none"> <li data-bbox="436 566 1436 599">1. (05mins) <u>Daily Rundown + Where we were/Where we are/Where we’re going</u> <li data-bbox="436 607 1923 639">2. (30mins) <u>Pearls Before Swine</u>: Play “Nim” game and hint at backwards induction. Introduce the concept of game trees. <li data-bbox="436 647 1923 712">3. (30mins) <u>Large Matrices Practice Problems + Review</u>: Wrap up work on larger matrix games with final set of exercises, run bounties on various problems and enrichment exercises for more difficulty. <li data-bbox="436 721 1948 753">4. (45mins) <u>Mixed Strategies and Unpredictability</u>: Explain the importance of playing truly unpredictably in mixed strategies <li data-bbox="436 761 1892 859">5. (90mins) <u>Games Against Nature</u>: Continue thought processes from “Jamaican Fishing” activity and introduce the “Umbrella Problem.” Students will work through creating their own utility numbers and move onto various ways of analyzing a game against nature.
	Afternoon	<ol style="list-style-type: none"> <li data-bbox="436 896 1944 961">1. (30mins) <u>Games Against Nature, continued</u>: Finish explanations and debates on how best to model games against nature with unpredictable outcomes <li data-bbox="436 969 1955 1034">2. (60mins) <u>Newcomb’s Problem and Free Will</u>: Discuss Newcomb’s Problem, allow class to debate, introduce the difficulty of prescribing exactly one strategy from the game theoretic perspective. <li data-bbox="436 1042 1919 1107">3. (30mins) <u>Failures of Game Theory</u>: Explain the shortcomings of applying to Game Theory, blindly, in all situations of strategy. Videoclips from Princess Bride and baseball game
	Evening	<ol style="list-style-type: none"> <li data-bbox="436 1156 1948 1253">1. (30mins) <u>Failures of Game Theory, continued</u>: Review what sorts of games we can solve, what games we can’t, and discussion on what situations would cause a failure of game theoretic analysis (e.g. not fully knowing the game, not know payoffs, etc.) <li data-bbox="436 1261 1898 1326">2. (45mins) <u>Permutations and Combinations</u>: Review Permutations and Combinations solving and equations. Return to Yahtzee solving for probabilities and expected value of a roll <li data-bbox="436 1334 1787 1367">3. (45mins) <u>Permutations and Combinations Exercises</u>: Exercises for permutations and combinations practice.

Day	Session	Activities
05	Morning	<ol style="list-style-type: none"> (05mins) <u>Daily Rundown + Where we were/Where we are/Where we're going</u> (60mins) <u>Probability Extension: Bayes' Rule</u>: Real-world applications of probability calculations (30mins) <u>Bayes' Rule Practice</u> (90mins) <u>Project Work</u>: Students go to computer lab and select articles on real-world situations of game theory being covered in contemporary media - prepare presentations to be done over the next week
	Afternoon	<ol style="list-style-type: none"> (30mins) <u>The Birthday Problem</u>: Probability, permutations, combinations application (60mins) <u>Expected Value Speaker</u>: Dennis speaks about Keno - expected value, permutations, combinations
	Evening (Sunday)	<ol style="list-style-type: none"> (15mins) <u>Project Work</u>: Students finish projects and ready presentations; first presentation (30mins) <u>Bayes' Rule Practice Review</u> (60mins) <u>Risk</u>: Students will play several rounds of Risk in small groups. Emphasize that they will need to take a "snapshot" of a strategic moment and try to model out both the precise Normal Form matrix for it, as well as any relevant probabilities and expected values for that moment. (30mins) <u>Risk Modeling and Discussion</u>: Students will share their "snapshots" of strategy and explain their optimal
		decisions at each juncture. See if we can hatch together a stronger mode of playing Risk given the aggregated strategies.

WEEK TWO: Complicating Models with Real-World Scenarios and Non-Zero-Sum Games

Day	Session	Activities
06	Morning	<ol style="list-style-type: none"> (30mins) <u>Iterated Games</u>: Discussion of how playing a game against the same opponent can change overarching strategy outside of a single game solution (30mins) <u>Game Tree Introduction</u>: Introduce concept of non-simultaneous games to students (zero-sum games) (90mins) <u>Game Trees and Competitive Decision Making</u>: Extension onto extensive form games (30mins) <u>Zero-Sum Game Wrap-Up</u>: Discuss what is capable of being modeled in zero-sum formats and bridge into non-zero-sum analysis
	Afternoon	<ol style="list-style-type: none"> (15mins) <u>Article Presentations</u>: Students continue article presentations (60mins) <u>Two Person Non-Zero-Sum Game</u>: Introduce first model of two-person non-zero-sum simultaneous games. Remind students that we are returning to basic restrictions and will continue to complicate from there (60mins) <u>Nash Equilibria</u>: Concepts of solutions and transition from descriptive models to prescriptive models given the method of solving for Nash Equilibria
	Evening	<ol style="list-style-type: none"> (45mins) Reading (30mins) Reading Discussion (45mins) <u>The Kidnapping Game</u>: Bridge between extensive form games and non-zero-sum situations

Day	Session	Activities
07	Morning	<ol style="list-style-type: none"> (15mins) <u>Article Presentations</u> (30mins) <u>Text Reading and Problems</u>: Exercises for Battle of Sexes/Chicken/Stag Hunt/Evolutionary Stable Strategies and analyzing which is which in exercises (30mins) <u>Coordination Game</u>: Model coordination game with lack of communication between players, leading well into the “Prisoner’s Dilemma” when no communication is allowed (45mins) <u>The Prisoners’ Dilemma</u>: Run through Prisoner’s Dilemma exercise to introduce touchstone model of Game Theory (30mins) <u>Battle of the Sexes</u>: Basic models of cooperation and competition, along with analysis of solutions and applications
	Afternoon	<ol style="list-style-type: none"> (45mins) <u>Chicken and Stag Hunt</u>: Basic models of cooperation and competition, along with analysis of solutions and applications
		<ol style="list-style-type: none"> (30mins) <u>Classic Non-Zero-Sum Game Worksheet</u>: Students work through classifying a number of games based on the staple forms of non-zero-sum games (30mins) <u>Evolutionarily Stable Strategies</u>: Return to video of colored-lizards; discuss relationship to rock-paper-scissors and show connection to Biology via Evolutionary Stable Strategy analysis
	Evening	<ol style="list-style-type: none"> (30mins) <u>Evolutionarily Stable Strategies, Continued</u>: Run through additional examples and complications, as well as discussion, of evolutionary models (60mins) <u>Strategic Moves</u>: Model coordination games with communication between players (especially Mutually Assured Destruction and other commitment devices) (30mins) <u>Repeated Prisoner’s Dilemma</u>: Tournament for Repeated Prisoner’s Dilemma algorithms - see which students can create the greatest expected value for an algorithm in RPD
08	Morning	<ol style="list-style-type: none"> (05mins) <u>Daily Rundown + Where we were/Where we are/Where we’re going</u> (170mins) <u>Movie - Thirteen Days</u>: Explain activity to students. Watch movie around the Cuban Missile Crisis - students will pick a “Snapshot” of the movie to model in-depth and analyze for strategic moves discussed in previous classes. (30mins) <u>Joint Discussion of Thirteen Days</u>: Discussion with other Game Theory class on Thirteen Days - what situations presented themselves, attempt mini-models of game trees, important insights from movie
	Afternoon	<ol style="list-style-type: none"> (15mins) <u>Article Presentations</u> (30mins) <u>Discussion of Thirteen Days</u>: Combine models of strategic situations from the movie for students in hyper-complicated game tree. Discussion on Game Theory’s applicability to real-world events given the apparent complexity (45mins) <u>Modeling for Thirteen Days</u>: Students will explain and analyze the situations they found for “Thirteen Days” via a written assignment to better practice articulating their thoughts to the layman advisee

Day	Session	Activities
	Evening	<ol style="list-style-type: none"> (30mins) Modeling for Thirteen Days, Continued: More writing time for their assignments, share-out and final discussion (45mins) <u>Text Reading</u>: Readings on Brinkmanship and Unpredictability to better apply Cold War game theory to other situations including political gridlock (45mins) <u>Evolutionarily Stable Strategies and Strategic Moves Practice</u>: Exercises on both topics to ensure practical understanding for solving both methods
<u>09</u>	Morning	<ol style="list-style-type: none"> (05mins) <u>Daily Rundown + Where we were/Where we are/Where we're going</u> (15mins) <u>Article Presentations</u> (20mins) <u>ESS and Strategic Moves Practice Review</u>: Review problems from previous night (30mins) <u>N-Person Games</u>: Movement away from 2x2 models of games to n-player games and crowd strategy/psychology (120mins) <u>Trust Game</u>: Connect from trust necessary in Thirteen Days to trust game when players are more selfish, more rational, and with less communication
	Afternoon	<ol style="list-style-type: none"> (15mins) <u>Article Presentations</u> (45mins) <u>Trust Game Discussion</u>: Wrap up "Trust Game" with final discussion and any applicable insights to larger games (45mins) <u>The Travelers' Dilemma</u>: Modification of Prisoner's Dilemma
	Evening	<ol style="list-style-type: none"> (45mins) <u>The Pirate Game</u>: Connection to <i>The Dark Knight</i> and discussion of voting in contexts of bribery or immediate monetary gain, students attempt to solve for solution to 10 player version (30mins) <u>Text Reading and Problems</u>: Expansion on Pirate Game applications and text-reading from Dixit/Nalebuff and contemporary news sources (30mins) <u>Three-Person, Two-Strategy Game</u>: Students play and model a three-person, two-strategy game as an introduction to formally solving n-player games
<u>10</u>	Morning	<ol style="list-style-type: none"> (05mins) <u>Daily Rundown + Where we were/Where we are/Where we're going</u> (45mins) <u>Utility Theory</u>: Discuss methods of deriving payoffs for models using Utility Theory (45mins) <u>Pimslers Dementia and Tragedy of the Commons Demonstration</u>: Introduction to Tragedy of the Commons - another, advanced model of N-player games with high applicability to real-world scenarios. Demonstration activity in class for "Candy Tree" (90mins) <u>Tragedy of the Commons Reading and Discussion</u>: Students read Dixit/Nalebuff for theorizing of importance of Tragedy of Commons, discussion on role of morality and social benefit in the realm of Game Theory (60mins) <u>The Other Person's Envelope is Always Greener</u>: Activity requiring strategy and leading from games of perfect information to games of hidden information in non-zero-sum context. Lead-in to Signalling games.

Day	Session	Activities
	Afternoon	1. (120mins) <u>Signalling Games</u> : Discuss complicated models of incomplete information and hidden information in situations of strategy and communication. Play “The College Game.” Begin with games and move into analysis, solutions, and optimal strategies/applications
	Evening (Sunday)	1. (30mins) <u>The Volunteers’ Dilemma</u> : Extension of Tragedy of the Commons 2. (20mins) <u>Bystander Effect and Kitty Genovese</u> : Extension of Volunteers’ Dilemma and connection to psychology 3. (15mins) <u>Final Project Outline</u> : Outline of final project for students and introduction to expectations 4. (60mins) <u>Project Brainstorming and Research</u> : Bring students to computer lab after brainstorming possible situations and assignment of groups

WEEK THREE: Crowd Strategy and Creating Your Own Models

Day	Session	Activities
11	Morning	1. (120mins) <u>Final Project Work Session</u> : Students will work on final project models and create powerpoints/posters 2. (90mins) <u>Fair Division</u> : Introduce concept of fair division to students and solve basic fair division models. Discuss basic terms and have students attempt to intuit solutions
	Afternoon	1. (45mins) <u>Splitting a Cake</u> : Order cake from dining, students decide on a number of optimal strategies by which to fairly (or unfairly) divide the cake 2. (45mins) <u>Discrete Division</u> : Work through examples of unsplitable items and splitting them fairly using the Sealed Bids auction method
	Evening	1. (30mins) <u>Fair Division Practice</u> : Exercises and worksheet review for fair division methods after application 2. (60mins) “Avalon”?: Students play and analyze game of “Avalon” to discuss imperfect information, signaling, probability assessments, and so on 3. (30mins) <u>Avalon Analysis</u> : Students discuss and digest strategy in Avalon and how it may apply to more common scenarios of mistrust, deception, and business
12	Morning	1. (45mins) <u>Coalitions</u> : Introduce game of Coalitions - group alliance strategy and voting strategy 2. (60mins) <u>Voting Strategy</u> : Group strategy with lack of communication and with “positioning” models for politics 3. (60mins) <u>Voting Power</u> : Introduce formal methods of calculating voting power and coalition power given positioning models (Banzhaf index)

Day	Session	Activities
	Afternoon	<ol style="list-style-type: none"> (45mins) <u>Voting Power Exercises</u>: Students practice going through voting power calculations and strategy, as well as side-payments method of defining equilibria (30mins) <u>Positioning Games</u>: Model voting and partisanship through the positioning model and compare to real-world analysis and insights
	Evening	<ol style="list-style-type: none"> (45mins) <u>Coalitions</u>: Continue game of Coalitions with more advanced setups requiring deeper calculations and agreements (45mins) <u>Dollar Auction</u>: Modeling rationality in competition with one another for second-pay sealed bid auctions
13	Morning	<ol style="list-style-type: none"> (60mins) <u>Auctions</u>: Begin end-of-session auctions for students to use up their classroom currency, introducing classic bid, sealed bid, English auction, etc. (60mins) <u>Final Project Worksession</u>: Students completely finish their presentations and models to be presented beginning the next class
		<ol style="list-style-type: none"> (60mins) <u>Final Exam Review</u>
	Afternoon	<ol style="list-style-type: none"> (30mins) <u>Limitations of Game Theory, Revisited</u>: Cover what we can reasonably and confidently do with Game Theory, and what continues to fall outside the bounds of our field (90mins) <u>Introduction to Behavioral Economics</u>: Discuss the prospect of irrationality as playing a large role in economics, game theory, and all other human sciences. Begin basic modeling of irrational players and overview of important biases (Confirmation Bias, Halo Effect, Prospect Theory, etc.)
	Evening	<ol style="list-style-type: none"> (120mins) <u>Post-Test/Final Exam</u>
14	Morning	<ol style="list-style-type: none"> (30mins) <u>Student Program Evaluations (SPE)</u> (60mins) <u>Final Project Presentations</u>: Students present their final projects to the class and field critiques and questions (60mins) <u>Auction Theory, Pt. 2</u>: Finish final few models of auction modeling, particularly with penny auctions and qubit auction
	Afternoon	<ol style="list-style-type: none"> (60mins) <u>Final Project Presentations</u>: Students present their final projects to the class and field critiques and questions (60mins) <u>Analysis/Playing of Constructed Games</u>: Students play and critique games of others, searching for game solutions or equilibria (15mins) <u>RPS Tournament, Pt. 3</u>: Return to most basic model of game theory we began with, sufficiently complicated by our study of theory in repeated play, mixed strategies, strategic moves, and communication

Day	Session	Activities
<u>15</u>	Morning	<ol style="list-style-type: none">1. (05mins) <u>Daily Rundown</u> + <u>Where we were/Where we are/Where we're going</u>2. (60mins) <u>Final Auction</u>: Students use up all their classroom winnings on dollar auctions and final runthrough of auction theory3. (60mins) <u>Game Theory for Life</u>: Discussion on applications to Game Theory in everyday life and how this study and practice can be continued outside the classroom for your benefit and the benefit of others