

Inductive and Deductive Reasoning

CTY Course Syllabus

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[Day 4](#) (Axiomatic Systems)

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Inductive and Deductive Reasoning Curriculum

Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

CTY

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 1 (Recursive Formulas and Inductive Reasoning) Objectives**

- Cover Class Introductions, Rules, Expectations
- Find and write recursive formulas from number sequences.
- Define and give examples of inductive reasoning.

Morning

Activity	Time
Seating	5
Cake-cutting Problem (01B, 01B) Instructor and TA circulate, giving pointers, answering questions. Stop whenever it seems appropriate, but before people have made too much progress	15
Introductions, Rules, Honor Code <ul style="list-style-type: none"> • Instructor and TA introduce themselves (5 min) • Birthday Game (5 min) • Scavenger Hunt (01C), 10 min • Students makeup classroom rules (add rule that says you must be working on classwork at all times, if done early, you need to ask TA or instructor for next assignment) • Course Handout (00B) • Review Honor Code, Have students sign paper 	40
Chart paper with honor code on it, markers	
Pre-Assessment Test (01D)	15
Break	5
Introduction to Recursive Formulas <ul style="list-style-type: none"> • Definition of Sequence, terms, variables (5) • Example 1 (5) • Create own sequence and answer example questions (10) • If time permits: have a couple of students put their sequences on board and rest of class answer questions about it (10) • Definition of Recursive formula, Ex. 1 (10) • Example 2. If students finish early, they can write formulas for their sequences from before 	30
Recursive Formula Activity <ul style="list-style-type: none"> • Introduce, give example (5) • Have students prepare formulas and tables. Circulate to check that students are writing with correct notation (10) • Students alternate writing on board and calling on students for formulas; give first three rows first and then add more as necessary. Each student should take about 2-3 minutes (35). Have students write down each sequence and formula. 	50
Cake Cutting, Part II <ul style="list-style-type: none"> • Finish up table, write recursive formula for this situation. Call it C_n • Early finishers: make neat drawings of slices 4, 5, 6, ... 	15

Day 1: Recursive Formulas and Inductive Reasoning

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 1 Afternoon**

Activity	Time
<p>T.A: Lead in Reading The Number Devil Chapter 1: This shows the number devil trying to make all numbers by multiplying numbers of 1's:</p> $11 \times 11 = 121$ $111 \times 111 = 12,321$ $1,111 \times 1,111 = 1,234,321$ <p>Have students write down the given products, and have them guess what the products of $111,111 \times 111,111$ and $1,111,111 \times 1,111,111$ will be.</p> <p>At the end, explain</p>	15
Palindromes	5
<p>Inductive Activity: What is $11,111,111,111 \times 11,111,111,111$?</p> <p>Patterns do not always hold; induction does not always work. Have students do by hand and compare answers (123,456,790,120,987,654,321)</p>	20
<p>Inductive Reasoning Lecture</p> <ul style="list-style-type: none"> Define induction, give crows example (5) Relate induction to recursion (5) Shortfall of incursion: it does not always work! (5) 	15
<p>Traffic Jam (01E)</p> <p>Will need chalk or masking tape</p>	20

Evening

Activity	Time
Thought Question (write & discuss)	10 + 10
Recursive Formulas Assignments	45 – 60
<p>Daily letters:</p> <p>Introduce concept – an opportunity to tell us anything on your mind. You will generally get prompts and suggested topics, but if you want to write about other things as well, that would be great! We will read and respond to them each day.</p> <ul style="list-style-type: none"> “How was your day? Which activities did you particularly like/dislike?” What do you hope to learn this summer? What do you think your friends like about you? Anything else? 	10
<p>Supplementary Activities</p> <ul style="list-style-type: none"> Coin-Weighing (S02). Need pennies. Polygon Corners (S03). Need scratch paper, scissors, tape. 	

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Day 2 Afternoon

Activity	Time
T.A: Lead in Reading The Number Devil Chapter 2 <ul style="list-style-type: none"> - Roman Numerals: I, C, X, L, C, M, D - Write some simple numbers like 1 – 9. How would you write 40? 50? 60? 90? 400? 900? - What number is MCMLXXXVI (p. 41) - Write your birthday in Roman Numerals - Write your birthday using powers of 10 	30
Finish CrissCross Cubes	30
Finish Traffic Jam	15
If extra time: Hexagonal Numbers Introduce; ask if anyone would like to try drawing on board. <div style="text-align: center; margin: 10px 0;"> <p>Formula: $3n(n-1) + 1$</p>  <p>1 7 19 37</p> </div> Make table, Recursive formula: $h_1 = 1; h_n = h_{n-1} + 6(n - 1),$ Explicit: $h_n = 3n(n - 1) + 1$	

Day 2 Evening

Activity	Time
Thought Question (Circular Numbers); discuss tomorrow <i>We have studied triangular, square, and other kinds of numbers. Could there be circular numbers? Explain.</i>	15
Explicit Formulas Assignment	60
Daily Letters <ul style="list-style-type: none"> • What do you like most/least about being away from home? 	10
Supplemental Activities Penny Problem (S01) . Needs Pennies Function Machine Game (Class activity)	

Inductive and Deductive Reasoning Curriculum

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Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 3 (Deductive Reasoning) Objectives**

- Understand deductive reasoning and be able to distinguish it from inductive reasoning, given examples.
- Solve various deductive problem types.
- Distinguish truth from validity in simple deductive arguments.
- Recognize and determine the validity of syllogisms.

Morning

Activity	Time
Seating: Numerical order by number of letters in full name	5
Warm-Up, Review Difficulties from Explicit Formulas Assignment	25
Circular Numbers Discussion (thought question from yesternight)	20
Introduction to Deductive Reasoning <ul style="list-style-type: none"> • Census example (inductive?) explain why not – does not rely on a pattern formed from prior examples (5) • Definitions, Examples – student elicited (10) 	15
As class, Mr. Brown, Black, Blue example	15
Break	15
As a class: Softball Game (03B) , Tennis ball	30
Groups: Census-Taker Problems (03A)	55

. Day 3 Afternoon

Activity	Time
Number Devil Chapter 3 <ul style="list-style-type: none"> • T.A: Lead in Reading Chapter (15) • Define prime numbers (5) • 03C: Prime Number Sieve (up to 100) (10) • Goldbach: Couple of examples as sum of primes (10) $10 = 3 + 7$ $43 = 2 + 41$ $80 = 73 + 7$ 	40
Introduction to Syllogism <ul style="list-style-type: none"> • Definition, example (5) • 03D: Syllogistically speaking (individual, then review (10 + 5) • Valid v. invalid (5) • 03E: True/False handout (10) 	35

Day 3 Evening

Activity	Time
Thought Question – Bad Day A boy is having a bad day. He thinks to himself, "Every time I talk to my grandmother, I feel better. I'll give her a call." What kind of reasoning is he using? Inductive, deductive, both or neither? Explain.	10
03G: Inductive or Deductive Arguments	10
03H: Syllogism handout	20

Day 3: Deductive Reasoning

Inductive and Deductive Reasoning Curriculum

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Continue 03A: Census-Taker Problems; write up individually	30
Daily Letters	10

Inductive and Deductive Reasoning Curriculum

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Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 4 (Axiomatic Systems) Objectives**

- Use a system of axioms to reach valid conclusions and analyze the validity of conclusions.
- Determine whether syllogisms are valid or invalid.

Morning

Activity	Time
Seating: Numerical order by birthday (day only, ignore month and year).	5
Warm-Up: Which two primes can you add to give a sum of 88? Create a valid syllogism that has something to do with something you find interesting. What is the difference between deductive and inductive reasoning? Come up with an explicit formula for ...	15
Review Census Taker Homework; display good answers	20
Discuss thought question	5
04A Truth, Validity and Soundness (students take turn reading after charts)	25
Break	10
More syllogism Pass back syllogism HW, Review confusing ones, allow students to finish or ask about ones they got wrong.	20
Introduction to Axiomatic Systems TA organizes 04B Matrix Height Problem (need 1 copy). The ten names are cut up and distributed randomly to ten students. If there are any remaining students, they are each given some of the clues. The clue holders (students or TA) read the clues, and the students with names try to line themselves up in the front of the classroom according to the clues. Afterward, TA or Instructor discusses the notion of having a system of axioms (the clues) and making conclusions based on them (the final order). Relate this to deductive reasoning.	20
Break	5
04C BACO Axioms These are inspired by the CTY class that studies the Chesapeake Bay. Read over the axioms together, for example, by going around having each student read one sentence. Then have the students start on the problems individually. After everyone has finished problem 1, go over it together. You might have the entire class read it out loud in unison, with the answers filled in. Discuss how this should serve as a model for how they are going to answer each of the four parts of problem 2. Then give them time to work on problems 2 and 3. Check quality of work before accepting work from early finishers. This is a good opportunity for students to practice proving their conclusions using sound reasoning.	55

Inductive and Deductive Reasoning Curriculum

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Morning (8:45 – 11:45, 180 min)

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Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 4 Afternoon**

Activity	Time
Number Devil Chapter 4 TA leads reading. Explain graphical demonstration about diagonal of a unit square. Have unit squares (1 black square, two red squares per student) on construction paper, students cut into triangles to see demonstration -	30
04D Lady/Tiger Problems Students work individually or in pairs. Encourage students to check their answers with instructor/TA every few problems. They do not have to justify or explain their work in writing, but encourage them to explain it to each other and/or to you.	45 – 75
Have the students explain why these problems are an example of an axiomatic system. Also, point out to the students that they are solving the problems by the process of elimination. When all other options have been shown to be impossible, the one remaining option must be correct. (This is similar to a proof by contradiction, a concept which can be presented later.)	

Day 4 Evening

Activity	Time
Thought Question (04E) What would be sufficient evidence to prove to you that there was life on another planet? What do you think would convince a scientist? Explain. Discuss tomorrow.	15
Matrix Problem Packet (04F) Students work individually (or in pairs) on packet of matrix problems. Review an example as a class first.	65
Daily Letters	10
Supplemental Activity (S05) Dames for Delphi; Freida's Fiance	

Inductive and Deductive Reasoning Curriculum

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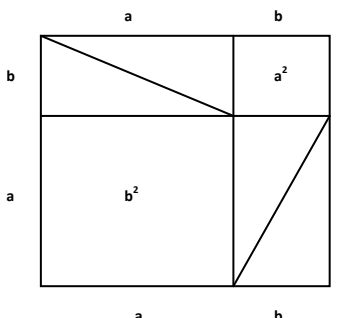
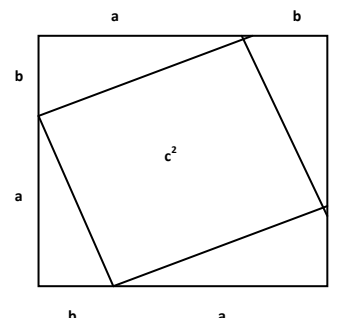
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Day 5 (Proofs) Objectives

- Explain simple graphical and algebraic proofs.

Morning

Activity	Time
Seating: between two people next to whom you have not yet sat	5
Warm-Up Based on your reading from <i>The Number Devil</i> and your own thoughts, do you personally believe that 0.999... repeating forever does, in fact, equal 1? Explain.	5
Discuss Warm-Up	10
Discuss Thought Questions (Levels of Proof)	15
<p>Introduction to Graphical Proofs and the Pythagorean Theorem</p> <ul style="list-style-type: none"> • Prove: <i>The length of a diagonal of a one unit square is root 2.</i> • Show sqrt of 2 movie as review of yesterday (5) • Discuss how to formally write the proof (5) <i>We can see from the diagram that the diagonal of a unit square is the side of another square. This new square is made of four equal triangles, each with area of ½. So the new square has an area of 2. Thus, by our premise, the length of each side of the new square must be the square root of 2.</i> • Challenge: Graphically demonstrate that the area of a triangle equals ½bh. (10) • Introduce Pythagorean Theorem: On graph paper, students draw three or four right triangles and measure their side lengths. Record data on board, give formula and test (15) • Show graphical proofs of Pythagorean theorem (5) • Walk through written proof of Pythag, on graph paper (15) <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>Note that each diagram is a square with side length $a + b$ and area $(a + b)^2$. Thus, both squares have the same area. Each of the 4 triangles in each diagram have the same area ($\frac{1}{2} ab$). So, we can subtract the four triangles from each diagram and the remaining areas are still equal. Therefore, $a^2 + b^2 = c^2$.</p> <p>Rulers, graph paper, colored pencils</p>	55
Break	15
05A Graphical Proofs Assignment (will finish later in the afternoon)	25
Coalitions need poker chips, playing cards	50

Day 6: Limits & Fibonacci Numbers

Inductive and Deductive Reasoning Curriculum

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Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

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Day 5 Early Afternoon

Activity	Time
Number Devil Chapter 5 (figurate numbers) led by TA	20
Graphical Proofs Assignment *early finishers can do challenge questions	30
<p>Algebraic Proofs that $0.\overline{9} = 1$ <u>Algebraic proofs</u> demonstrate the conclusion algebraically.</p> <p>Note that a simpler proof that $0.999 \dots = 1$ is: $1/3 = 0.3333\dots$ $2/3 = 0.6666\dots$ By adding these we get</p> <hr/> $3/3 = 0.9999\dots = 1$ <p>Proof that $0.9999 \dots = 1$:</p> <p>Let $x = 0.9999 \dots$ multiply both sides by 10</p> <p>$10x = 9.9999 \dots$ subtract x from both sides $-x = -0.9999 \dots$</p> <p>$9x = 9$ divide both sides by 9 $x = 1$</p> <p><i>Some students should note that this essentially begs the question. Now we should challenge whether $0.3333\dots$ really equals $1/3$.</i></p> <p><u>Challenge students to use this method to show that $1/3 = 0.3333 \dots$ or harder ones...</u></p> <p><i>Discuss whether the students were convinced by these proofs. Do they now believe $0.999\dots = 1$ or not?</i></p>	35

Day 5 Evening

Activity	Time
Finish graphical proofs/Complete unfinished assignments	70
Daily letters	10
If lots of time left, play coalitions, if a little time, can play wink murder or spoons. Need plastic spoons	

Day 6: Limits & Fibonacci Numbers

Inductive and Deductive Reasoning Curriculum

CTY

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Day 6 (Limits & Fibonacci Numbers) Objectives

- Apply a general understanding of the concept of limits and use relevant notation.
- Explore the Fibonacci sequence, golden ratio and their applications.

Morning

Activity	Time
Seating: Numerical order by street address	5
Warm-Up: A graphical proof of area of a trapezoid, rhombus	15
HW Review: Graphical Proofs challenging problems	15
Introduction to Limits <ul style="list-style-type: none"> • Definition of limit (2) • Example 1: students compute and share terms, point out notation, how to say (8) • What kind of reasoning is used to find a limit? (Inductive) • Example 2: students compute partial sums to guess limit. Notation, pronunciation (5) • Example 3 (5) • Example 3 Graphical proof, students copy down (5) 	25
Break	10
Introduction to Limits, Cont <ul style="list-style-type: none"> • Distribute squares of construction paper or color copier paper. Have students cut off corners, ensuring not to cut off too much or too little. Should be able to go up to 9 or 10 sides. Show video for what happens as n approaches infinity. (15) Need color paper cut into large squares, scissors • Example 5 & Proof (15) • Convergence & Divergence (5) 	35 105
Fibonacci Numbers and the Golden Ratio <ul style="list-style-type: none"> • Define Fibonacci numbers; have students find first 17 (10) • Ratio sequence for Fibonacci, and limit (10) 	20
Break	5
Fibonacci Spirals <ul style="list-style-type: none"> • Give demonstration of drawing on slides (5) • Students work using graph paper, colored pencils, tape (15) 	30
06C Fibonacci and the Human Body <ul style="list-style-type: none"> • Either in pairs or alone, students record measurements of various body parts and find ratios; students write answers on board and find average (20) Need tape measures • Fibonacci videos (10) 	20
OR Stick a student to the wall activity (requires 3 rolls of duct tape , student in long sleeves)	

Inductive and Deductive Reasoning Curriculum

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Day 6 Afternoon

Activity	Time
The Number Devil Chapter 6 (Fibonacci Numbers), led by TA Ensure that students understand the rabbit example and the table.	20
06A Golden Rectangle and Spiral, 06D Need compasses and rulers	20
06E Honeybee Family Tree Need colored pencils	20
Fibonacci Patterns Create a table of the first 30 numbers in the Fibonacci series. In different colors, shade in the multiples of 2, 3, 5, 8. Does a pattern emerge? Need colored pencils	15

Day 6 Evening

Activity	Time
06B Worm Problems	40
Thought Questions – discuss if time permits 1. [Have picture drawn and labeled) What shape is the limit of a cone whose base stays constant, but whose height approaches infinity? Explain. Would approach a cylinder of infinite height- sides go to being nearly parallel, top point becomes a line) 2. What is the limit of $1/x$ as $x \rightarrow 0$? Explain. (Hint: use numbers that are smaller and smaller, closer to 0 to determine what happens, like $x = 10, 1, .1, .01, .001, \dots$) On discussion, show how to divide these numbers if they were fractions.	15 + 20
Daily Letters	10

Supplemental Activities

S06 Fibonacci Puzzles

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 7 (Symbolic Logic) Objectives**

- Construct truth tables using not, and, or, if-then, if and only if, and combinations of these connectives.
- Translate English into symbolic logic and vice versa.
- Experience some of the patterns in Pascal's triangle.

Morning

Activity	Time
Seating: Alphabetical order by last name spelled backwards.	5
Warm-Up: List three patterns/ interesting things about the Fibonacci numbers. Find the limit as $x \rightarrow 0$ of ...	15
HW Review: Third problem, 2 nd problem, discuss thought question if not done so last night	10
Snowflake Drawing <ul style="list-style-type: none"> • Show example on slides (5) • Students make own snowflakes (07B) (15) • Talk about limits of area, and perimeter (perimeter limit does not exist, as perimeter multiplies by 4/3 each time. (10) 	30
Break	10
07A Introduction to Symbolic Logic <ul style="list-style-type: none"> • Symbolic Logic definition, examples (5) • Definitions of Statements P & Q (5) • Negation of P, examples (5) • Truth table definition • Conjunctions, examples, pictures, relation to truth tables (10) • Disjunctions, examples, pictures, relation to truth tables (10) • Conditional statements, examples, pictures, relation to truth tables (10) • Biconditional statements, examples, pictures, relation to truth tables (5) • Combinations (10) 	60
07D Logic Activity Group <ul style="list-style-type: none"> • Fill out tables on paper (10) • Put information on poster, plan presentation (10) • Presentations (10) 	30
07C Translating Symbolic Logic Students will finish in the afternoon or evening.	20

Inductive and Deductive Reasoning Curriculum

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Evening (4:00 – 5:30, 90 min)

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Day 7 Afternoon

Activity	Time
Number Devil, Chapter 7 , led by TA	20
Pascal's Triangle <ul style="list-style-type: none"> Copy down first few, observe patterns (check notes) Color odds and evens using 07E coloring template. Students choose one color for odds, another for evens. Color the top three hexagons the odd color, since they would be the number 1. Since $O + O = E$, $O + E = O$, $E + E = E$ to continue. (Need colored pencils, may continue later) 	20
Truth table Group Activity <ul style="list-style-type: none"> Students work together in groups, using paper squares to make truth tables on the classroom floor. 	25
Thought Question Sometimes people reach a conclusion after just one experience. ($A_1 = B$, therefore all $A = B$) When is generalizing from one example a <u>good</u> idea? When is it a <u>bad</u> idea? Explain. Use examples. (Discuss tomorrow.)	15

Day 7 Evening

Activity	Time
Continue Symbolic Logic Activities and/or Truth Tables	30
Mid-Course Self-Evaluation and Daily Letters Students write comments about their own performance in each of the areas. They are to write in bullet points or sentences, with examples if possible. Early finishers write a daily letter. <ul style="list-style-type: none"> Inductive reasoning (finding and extending patterns, finding formulas, limits, etc.) Deductive reasoning (syllogisms, matrix problems, axiomatic systems, symbolic logic, truth tables, etc) Solving other types of problems Writing clear solutions and explanations Writing answers to thought questions Participating in discussions/volunteering answers and ideas Listening to others' ideas Taking notes, including required and optional notes Neatness and organization of notes and other work Working alone Working with partners/in groups Trying your best, even when it is difficult Following directions, staying on task, using class time well Asking for help when you need it Anything else you would like to add 	30
Continue Symbolic Logic Activities and/or Truth Tables, 07G More Truth Tables	30

Day 7: Symbolic Logic

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Supplemental Activities

- [S07 Tautologies](#)
- [S08 Carpool Axioms](#)

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Day 8 (Symbolic Arguments & Paradoxes) Objectives

- Determine the validity of logical arguments presented in English or in symbols.
- Write the contrapositive of an if-then statement, in English or symbols.
- Articulate the conflicting lines of reasoning that emerge from paradoxes and to evaluate those lines of reasoning.

Morning

Activity	Time
Seating: Numerical order by the sum of birth date and month.	5
Warm-Up: Make a truth table for: $(P \wedge Q) \wedge (\sim Q \vee P)$ New York is a great city. Being in great cities makes me happy. I am happy. Conclusion: I am in New York. Do the first three statements imply the conclusion? Explain with a truth table.	20
Homework Review/ Questions from Class	15
Thought Question - Sometimes people reach a conclusion after just one experience. ($A_1 = B$, therefore all $A = B$) When is generalizing from one example a <u>good</u> idea? When is it a <u>bad</u> idea? Explain. Use examples. Some times when it is a good idea: <ul style="list-style-type: none"> • Any kind of safety issue. Should not wait until the second or third time you are hit by a car to decide to look before crossing a street, and many similar examples. • When repetition is impossible or impractical. For example, when studying global warming you cannot compare earth's climate with a control earth in which humans did not burn fossil fuels and raise the carbon dioxide level. Or when studying a certain kind of extinct animal, you may only have one fossil. • Matters of personal preference with low stakes. For example, "I went to one horror movie and I hated it, so I am not going to any more." (Horror movies are easy to avoid.) Some times when it is a bad idea: <ul style="list-style-type: none"> • Any kind of stereotyping based on race, appearance, etc. • Judgments of personal ability. For example, "I wasn't good at this the first time I tried it, so I am just no good at it and won't try again." • Matters of personal preference with high stakes. For example, "I ate one kind of vegetable once which I didn't like, therefore I will never eat any vegetables again." (Vegetables are hard to avoid and doing so is bad for your health.) • Scientific conclusions should be based on repeated experiments whenever possible. One time when it could be a good idea or a bad idea: <ul style="list-style-type: none"> • Trust issues. "This person was mean to me once, so I am never going to trust him/her again." Might be smart (especially if a stranger); might be too unforgiving (especially if a family member). 	15
Break	10
08A Introduction to the Contrapositive Circulate and help as needed. Discuss when appropriate. Have students give examples of if-then statements and their contrapositives.	15
08B Evaluating Symbolic Arguments	30
Break	10

Day 8: Symbolic Arguments & Paradoxes

Inductive and Deductive Reasoning Curriculum

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08C Symbolizing Arguments	30
08D Logic Puzzles (Group)	30

Day 8 Afternoon

Activity	Time
The Number Devil, 8th Night	20
08E Analyzing Paradoxes Read worksheet aloud together. Discuss enough to be sure students understand. I usually go over the first paradox and give students some time to answer those questions before I go over the last two paradoxes. Then I give students more time to work individually. Discuss it with the whole class tomorrow, to give students more time to think about it and give you time to read their ideas. (See the discussion points at the end of the assignment.) I often mark particularly good insightful student responses and tell students with responses marked that they need to explain them when we get to that question in the discussion. This helps insure that quiet students with good ideas speak up.	55

Day 8 Evening

Activity	Time
08F Painted Cube Project, Part I of II	80
Unit cubes	
Daily Letters	10

Supplemental Activities

S09 Knights & Knaves

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)

[Materials](#)

[Teacher Guide](#)

Day 9 (Reasoning Fallacies) Objectives

- Recognize and name common informal fallacies in written and oral arguments.

Morning

Activity					Time
Seating: Alphabetical by the name of the street on which you live					5
Warm-Up Translating sentences from English to symbols Contrapositives Paradoxes					20
Review Symbolic Logic worksheets, etc.					15
Review Objectives					
09A Introduction to Fallacies <ul style="list-style-type: none"> • Students read definition of fallacy • Give history of fallacies • Fallacies – 4-5 minutes each <ul style="list-style-type: none"> ⊕ Equivocation, Appeal to Authority, Irrelevancy, Messenger/Message, Ad Hominem, Slippery Slope, Weak Analogy 					30
Break					10
09A Introduction to Fallacies, cont. <ul style="list-style-type: none"> • Fallacies – 4-5 minutes each <ul style="list-style-type: none"> ⊕ Emotional Appeal, Questionable cause, Deceptive statistics, confident Speculation, begging the question, false dichotomy, can't back down 					30
Fallacies, fallacies, Fallacies!					
1) Quest.cause 2) Appeal to auth. 3) Quest. cause 4) False analogy 5) Slippery slope 6) Begs the q	7) Slippery slope 8) Quest. cause 9) False dichotomy 10)Begs the q 11)Quest, cause 12)False dichotomy	13)Slippery slope 14)Ad hominem 15)Quest. cause 16)Equivocation 17)Quest. cause 18)weak analogy	19)slippery slope 20>false dichotomy 21)deceptive stats 22)Appeal to auth. 23)Begs the q. 24)Quest. analogy	25)Appeal to emot. 26)False dichotomy 27)Equivocation 28)Appeal to auth. 29)Begs the q. 30)Ad hominem	50

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 9 Afternoon**

Activity	Time
Number Devil: The 9th Night Before reading the next page, have students identify rows 3 – 7 on p. 178 (primes, Fibonacci, triangular, powers of 2, factorials)	20
09B Fallacy Skits <ul style="list-style-type: none"> - 10 minutes to read out as a class - 20 minutes (or as long as it takes for each group to have completed at least one skit) - 20 minutes for each pair to act out skits, while others guess which fallacy is being acted out. <p>Note: an important part of this activity is that the skits include figuring out a productive way to respond to each fallacy. Discourage writing skits that consist of arguments that go nowhere with nobody listening to the other side and/or both people committing fallacies.</p> <p>Props for skits?</p>	55

Day 9 Evening

Activity	Time
Finish Painted Cube Projects Before reading the next page, have students identify rows 3 – 7 on p. 178 (primes, Fibonacci, triangular, powers of 2, factorials)	80
Daily letters	10

Suggested Supplemental Activities[S10 Magic Triangles](#)

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 10 (Strategies) Objectives**

- Generalize from the solution to a specific problem to a strategy to solve groups of related problems.
- Continue practicing recognizing and naming common informal fallacies in written and oral arguments.

Morning

Activity	Time
Seating: Somewhere you have not sat before	5
Warm-Up Identify the fallacies Choose three fallacies not listed above. Write your own example for each one. What is the remainder when 354 is divided by 23?	20
Wrap-Up From Yesterday Finish skits if not done yesterday, have students present painted cube problem parts if they want	10
Review Objectives	
Introduction to Strategies	5
10A Bottle-Counting Problems In pairs or small groups.	60
Break	10
10C Fallacy Identifications	60
Break	10
Coalitions Again! Have students remember their strategies from the last game, and decide what worked best.	40

Afternoon

Activity	Time
Number Devil Chapter 10	20
Fallacy Identifications, cont. When grading, star 1-2 good answers on each student's paper, ensure each question has been starred.	20
10B Light-Switching Problem Have students write up solutions neatly. Answer: Square Numbers are left on. Cubes, pennies, cards	35

Evening

Activity	Time
Finish light-switching problem, Complete incomplete HW, correct problems, etc.	70
Daily Letters	10

Supplemental Activities[S11 Perplexing Paths](#)[S12 Locker Problems](#)

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 11 (Abductive Reasoning) Objectives**

- Find modular arithmetic conversions of integers and add and subtract using modular arithmetic.
- Define abductive reasoning, compare it with inductive and deductive reasoning, and identify examples of all three.
- Identify reasoning fallacies used by debaters

Morning

Activity	Time
Seating: Numerical order by total number of siblings, including half and step.	5
Warm-Up: Random problem-solving riddles	20
Review Objectives	
Bottle Counting Review What strategies did students use?	10
Introduction to Modular Arithmetic <ul style="list-style-type: none"> • Definition (5) • Examples (10) • Real-life examples: time • Modular arithmetic as a circular number line (5) • More examples (15) • Generalizations (5) • Negatives (5) • Examples (15) 	60
Break	15
11B Introduction to Abductive Reasoning <ul style="list-style-type: none"> • Definition • Example • Other examples 	10
11C Blocker Game Reasoning: Butterflies need to be placed on any two squares of the same color. Each caterpillar covers a black and white square, so if butterflies are on same color, there are more black squares than white, or vice versa, so the caterpillars cannot cover them all. This is abductive reasoning. Need scissors to cut out the game pieces	35
Review Light-Switching Problem	10
S12 Locker Problem	

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 11 Afternoon**

Activity	Time
Number Devil: The 11th night	20
Fallacy Identifications Review	20
11D Debates Have students write debate topics on a slip of paper. Mix topics with provided topics. Select two volunteers. Choose a topic, who will be on which side, and who starts. Then let them debate until it seems to wind down. Call it off. Have debaters and audience comment on the debate – who was more convincing, what if any fallacies were used, feedback on poise, etc. Then select a new pair.	35

Day 11 Evening

Activity	Time
11E Thinking About Reasoning	40
11F Sticky Gum Problems	40
Daily Letters	10

Suggested Supplemental Activities[S12 Handshake Problem](#)

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 12 (Tower of Hanoi) Objectives**

- Recognize sound and unsound written and oral arguments.
- Articulate the two steps of formal inductive proof and why completing those steps constitutes a complete proof.

Morning

Activity	Time
Seating: In rainbow order by backpack color.	5
Warm-Up: Metaformulas	20
Review Work from yesterday - Sticky gum review	10
Review Objectives	
Thinking About Reasoning Discussion	20
12A Tower of Hanoi Game & Formulas Need construction paper squares of various sizes and colors I use purple 1 inch, blue 1 ½ inch, green 2 inch, yellow 2 ½ inch, orange 3 inch, red 3 ½ inch	35
Break	15
12B Introduction to Formal Inductive Proof Structure of Induction (5) Example 1: Tower of Hanoi (15) Example 2: Sum of consecutive integers (20) Practice: Sum of Squares, Sum of evens, inequality (30 + 10)	80

Day 12 Afternoon

Activity	Time
12C Hotel Infinity, Part I, led by TA	30
More Debates	45

Day 12 Evening

Activity	Time
12D Analyzing Arguments (in pairs) When grading, choose one or two from each student's homework to review tomorrow.	60
Daily Letters	10

Supplemental Activities[S13 Knights & Knaves II](#)

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)

[Materials](#)

[Teacher Guide](#)

Day 13 (Game Theory & Infinity) Objectives

- Articulate the strategies and pitfalls of a simple game of mutual cooperation or competition for mutual gain or loss.
- Articulate some of the real-world applications of game theory in politics, economics, etc.
- Distinguish between countable and uncountable infinities.
- Examine some of the seemingly paradoxical results of attempting to calculate with infinity.

Morning

Activity	Time
Seating: Add digits of zip code plus house #	5
Warm-Up: Are these arguments valid or not? Explain. <ul style="list-style-type: none"> - Only stupid boys like stupid girls. Sandy is a stupid girl, and Jeremiah likes her, so Jeremiah must be stupid. - Eric’s dad yells at Eric when he is mad. Eric’s dad is not yelling at Eric, so he is not mad. Prove by induction: For $n \geq 1$, $2 + 2^2 + 2^3 + 2^4 + \dots + 2^n = 2^{n+1} - 2$	25
Review Analyzing Arguments Worksheet	15
Objectives Review	
Paper-Scissors Game – do with another class, if possible <ul style="list-style-type: none"> - Introduce directions, scoring (5) - In pairs, students play until scoring is in a rut (5). - Switch partners and play two more rounds (15) - Discussion of strategies – what is your best strategy? (10) - Play two more rounds (10) - Discussion of strategies (10) 	55
Break	15
Game Debrief Prisoner’s Dilemma Description & Comparison (10) Applications (15) Tragedy of the Commons, examples (10)	35
Game Theory	15
Wink Murder	15

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 13 Afternoon**

Activity	Time
Hotel Infinity Part II	20
13A Infinity & Cantor’s String <ul style="list-style-type: none"> - Cantor, Kronecker Introduction (5) - Vocabulary: countable, set, element (15) - Showing countability (10) - Implications of Hotel Infinity (15) - George’s solution (10), need colored pencils - Infinity – infinity, undefined (5) 	60

Day 13 Evening

Activity	Time
Thought Questions: Game Theory What real-life situations are similar to the paper-scissors game? For each one you think of, briefly explain the similarities. What modern “commons” are being destroyed by the tragedy of the commons? Explain.	15
S14 Juggling Act	55
Daily Letters What, if anything, are you especially proud of that you would like mentioned in your evaluation?	10

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)

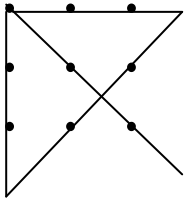
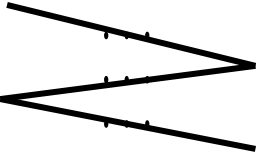
[Materials](#)

[Teacher Guide](#)

Day 14 (Reasoning outside the Box) Objectives

- Practice creativity in problem-solving.

Morning

Activity	Time
Seating: Anywhere	5
Warm-Up: Describe what is meant by Prisoner’s Dilemma and by Tragedy of the Commons. Why are they called “dilemma” and “tragedy”? Explain how steroid usage is an example of Prisoner’s Dilemma.	15
<p>Discuss Thought Question</p> <p>What real-life situations are similar to the paper-scissors game? For each one you think of, briefly explain the similarities.</p> <p>What modern “commons” are being destroyed by the tragedy of the commons? Explain.</p>	20
Objectives, Introduction to Reasoning outside the Box	5
<p>9-Dot puzzle</p> <p><u>Some possible solutions:</u></p> <p>4 lines:  3 lines: </p> <p>1 line:</p> <ul style="list-style-type: none"> • Place square on cylinder. Line wraps around. • Use one fat line, like a highlighter pen. <p>0 lines:</p> <ul style="list-style-type: none"> • Draw dots touching each other to begin with. • Fold paper so all dots are on top of each other. Stab a pin through them all. • Draw dots in crayon on wax paper. Melt with an iron 	30
Break	15
<p>12A Reasoning Outside the Box activities</p> <p><i>Needs toothpicks, ~45 cups</i></p>	90

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min)

Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 14 Afternoon**

Activity	Time
Lateral Thinking Puzzles & Story	35
Student Program Evaluations	20

Day 14 Evening

Activity	Time
Finish extra stuff <ul style="list-style-type: none"> - Observe class presentations - Prepare for closing day - 	80
Daily Letters	10

Inductive and Deductive Reasoning Curriculum

CTY

Morning (8:45 – 11:45, 180 min) Afternoon (12:45 – 2:00, 75 min)

Evening (4:00 – 5:30, 90 min)

[Handouts](#)[Materials](#)[Teacher Guide](#)**Day 15 (Closure)**

Activity	Time
Seating – wherever	
Logistics Clean classroom, pass back, discuss work, etc.	20
15A Reading List	10
Number Devil, Chapter 12	20
Certificates	10
Final letter “What have you learned at CTY which will be valuable for the rest of your life?” By articulating this they are more likely to make it true. Read their letters during break, or while they are signing yearbooks, or whatever, which give you a chance to write them any parting comments.	10