Probability and Game Theory
CTY Course Syllabus

DATE	ACTIVITY	CONCEPT
Sun. 6/27	Learn names; introduction to course, introduce the Battle of the Bismarck Sea as a 2-person zero-sum game.	
Mon. 6/28	**Morning:** Lecture: Review of Set Theory (Instructor - I) **Afternoon:** Lecture: Enumeration of sets (I) (Challenge Problem #1: generalize addition principle to the property of inclusion/exclusion) Lecture: Permutations and Combinations – including many examples (Teaching Assistant - TA) (Challenge Problems 2, 3 &4 on partitions.) **Evening:** Work quietly on first 2 written assignments. (TA)	-Sets, elements, subsets, universal sets, empty sets. -Set operations: union, intersection, complements, disjoint sets. - Properties of these operations: commutative, associative, distributive laws; DeMorgan's Laws. Venn diagrams - Addition principle. - Multiplication principle, Cartesian product of sets, power sets - Enumeration when order of selection matters and when it does not. Selection with and without replacement. - Partitions of integers - Sets and enumeration.
Tues. 6/29	**Morning:** Return and go over assignments (TA) Lecture: binomial theorem and set partitions (I) (Challenge problem #5: generalize the binomial theorem to a multinomial theorem.) Lecture: Introduction to probability theory (I)	 - Binomial coefficients; Pascal's triangle and properties. - Multinomial coefficients and enumeration of set partitions - Experiments, outcomes, sample spaces, events - Probability Models, probability of a union of events, probability of a

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		complement.
	Afternoon: Lecture: Uniform probability models (TA) **Quiz I** on sets and enumeration	- Uniform sample spaces, using enumeration to compute probabilities.
	Evening: Work quietly on assignments 3 & 4. (I)	Binomial theorem, probability.
Wed. 6/30	**Morning:** Quietly work on written assignments. Return and go over quiz. (I) Return and go over assignments (TA)	
	Lecture: Expected values/examples (I)	- Expected values - Independent Events
	(Challenge problem #6 – enumeration of compositions; challenge problems on probability: #7 – "Grid problem"; #8 – "Lemon candy problem" & #9 – Bertrand's Paradox)	- Ordered partitions (compositions)
	Lecture: Matrix algebra (I)	- Matrices, scaling, addition of matrices, dot product of vectors.
	Afternoon: Lecture: Matrix algebra, continued (I)	- Matrix multiplication, matrix determinants and inversion (2x2 case only), solving systems of linear equations.
	Class Discussion: Introduction to game theory – the resolution of the Bismarck Sea Battle (I)	- Game trees, game matrices, dominant strategies, minimax techniques and saddle points.
	Evening: Work quietly on assignment #5 Read Chapters 1-2 of text. (guest proctors – Academic Dean & Asst. Academic Dean)	- Expected values and matrix algebra.
Thurs. 7/1	**Morning:** Work quietly on assignments. Lecture: More on partitions (duality and Ferrer's diagrams) (I) Challenge Problem #10 – Putnam problem and #11 – Buffon's Needle.	- Infinite sample spaces; measuring sets.

DATE	ACTIVITY	CONCEPT
	Class Discussion: Strictly determined games. (I)	- More on dominance, saddle points and minimax techniques. - Higher order dominance - Movement diagrams.
	Afternoon: Class Discussion: Strictly Determined Games continued. (I) Video – "Zero-sum Games" from the 'For All Practical Purposes' series of educational mathematics videos.	- Value of a game, fair games, translating a game. Saddle points are equivalent and interchangeable. - Review of all concepts pertaining to strictly determined games - Introduction to non-strictly determined games. Repeated play and mixed strategies.
	Evening: Work quietly on assignment #6 Quiz II – binomial theorem, probability and expected values. Read Chapter 3 of text. (I)	- Strictly determined games, dominant strategies and saddle points.
Fri. 7/2	**Morning:** Work quietly on assignments. Return and go over quiz II. Lecture: Non-strictly determined games (I)	- Mixed strategies, probability vectors as strategies - Expected payoffs as matrix multiplication. - Expected Value Principle (when the opponent's strategy is known) - "Sol's Method" of equalizing expectation (when the opponent's strategy is not known) – 2x2 case only; - The Minimax Theorem
	Afternoon: Lecture: Non-strictly determined games, continued (I)	- Equalizing expectation in 2xn case or the mx2 case. Inactive strategies. - Equalizing expectation in the 3x3 case; Williams' theorem that for any game, the solution is the same as that of some square subgame. - Williams' method of oddments (2x2 case)
	Group Activity: Monte Carlo simulation	- Monte Carlo techniques,

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	of the Grid Challenge problem. (I & TA)	approaches to problem solving (trying small cases, pattern recognition, inductive reasoning…)
Sun. 7/4	**Evening:** Work quietly on assignment #7 (I & TA) Read Chapter 4 in text	- Non-strictly determined games
Mon. 7/5	**Morning:** Return and go over assignment #6 Work quietly on assignment #7 Lecture: Introduction to Linear Programming (I) **Afternoon:** Video: "Linear programming" Class Discussion: The Jamaican Fishing Problem (I) **Evening:** Work quietly on assignment #8 (TA) Read Chapter 5	 - Graphing and solving linear inequalities - Optimization; feasible and optimal solutions - Basic and non-basic solutions - Marginal values - Solving linear programming problems graphically - Application of game theory to Anthropology - Games against Nature - Chapter 4 - Linear Programming (graphical Method)
Tues. 7/6	**Morning:** Return and go over assignment #7 Lecture: Linear Programming (I) **Afternoon:** Lecture: The Simplex Method in Linear Programming (I) Lecture: Guerrillas, Police and Missiles (TA) **Evening:** Work quietly on assignments #8 & #9 (I)	 - Sensitivity Analysis - Slack variables, duality - Standard form, introduction to the simplex method. . - Simplex Method - Game theory in war situations - Interpreting payoffs as expected values - Chapter 5 - Linear Programming via simplex

DATE	ACTIVITY	CONCEPT
		method. - Duality
	Read Chapter 6 (optional)	- Application of game theory to Philosophy – the problem of free will
	Read Chapter 7	- Game Trees
Wed. 7/7	**Morning:** Quiz III	 - Matrix algebra and zero-sum games
	Lecture: Linear Programming (I)	- duality vs. phase I techniques for minimization problems
	Afternoon:	
	Lecture: Linear Programming applied to game theory (I)	- Solving mxn games via simplex method - Proof of the minimax theorem
	Lecture on game trees (TA)	- Normal form vs. extended form; game trees, information sets - Game trees, information sets, converting games from one form to the other
	Evening: Work quietly on assignments #8 & #9	-linear programming – solving dual problems
Thurs. 7/8	**Morning:** Lecture: Linear Programming and Games (I)	 - Solving zero-sum games via linear programming - Economic Interpretation of Duality
	Challenge problem #12 – a linear programming brainteaser	- Solving linear programming problems and games using Excel
	Afternoon: Lecture: Games in Extended Form (I)	 - Application of game theory to business - Games of partial information
	Lecture: Games Against Nature (TA)	- Playing against non-rational opponents - Axioms for playing games against

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	Evening: Assignment #10 and/or catch up on previous assignments Read Chapters 8 & 10	nature -Solving zero-sum games via linear programming -Games Against Nature
Fri. 7/9	**Morning:** Work quietly on assignment #10 Lecture: Conditional probability and Bayes theorem (TA) Class discussion on challenge problems **Afternoon:** Class Tournament Lecture: Introduction to Variable Sum Games (I)	 - probabilities of intersections of events - independent events - Illustrating Julia Robinson's theorem that repeated play using the expected value principle will converge to the optimal mixes - dominance, Nash equilibria - payoff polygons, Pareto Optimality - games solvable in the strict sense - Equalizing and prudential strategies
Sun. 7/11	**Evening:** Assignment #11 and/or catch up on previous assignments Read Chapters 11 & 12	 - variable sum games, payoff polygons, SSS games
Mon. 7/12	**Morning:** Work quietly on assignments Class discussion on dilemmas in variable sum games (I) Video: "Prisoner's Dilemma" **Afternoon:** Lecture: 2x2 Ordinal Games (I)	 - Examples: Prisoner's Dilemma, Chicken Dilemma - Iterated Games; 'Tit for Tat' - Applications – the superpowers arms race, Cuban missile crisis.

DATE	ACTIVITY	CONCEPT
	Evening: Assignment #12	- Nash equilibria, dominance, Prisoner's dilemma - Applications of the prisoner's dilemma
Tues. 7/13	**Morning:** Work quietly on assignments Lecture: Strategic moves (I) **Afternoon:** Lecture: Ordinal games and the Theory of moves. Applications to the Yom Kippur War. (I) **Evening:** Assignment 12 Read Chapter 14	 - Communications (threats and promises) - Sequential games; non-myopic equilibria - sequential games - n-person games
Wed. 7/14	**Morning:** Work quietly on assignments #11 & #12 Go over homework Class discussion on challenge problems. **Afternoon:** Class discussion on Evolutionary Stable Strategies (I) Introduction to n-person games (I) **Evening:** Read Chapter 15 Catch up on all assignments Fill out SPEs	 - Prisoner's dilemma in Puccini's opera "Tosca" - n-person version of Prisoner's dilemma; Tragedy of the Commons - Applications of game theory to biology - Evolutionary Stable Strategies - Examples of 2x2x2 zero-sum game with inequivalent and noninterchangeable Nash equilibria - Communications and coalitions; cooperative games
Thurs 7/15	**Morning:** Class Review of all topics;	

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	Study for final Exam Discussion of Challenge Problems **Afternoon:** Final Exam **Evening:** Movie: Thirteen Days (Cuban Missile Crisis)	
Fri. 7/16	**Morning:** Return Exams Hand out Solutions to Challenge Problems Class Party (bring games) **Afternoon:** Graduation	