Investigations in Engineering Self-Assessment - Solutions

Center for Talented Youth

Spring, 2011

1 Introduction

Here are solutions to the problems in the self-assessment. You may use these to check your answers and your work.

2 Functions

1)

$$\begin{aligned} Midpoint &= \left(\tfrac{x_1+x_2}{2}, \tfrac{y_1+y_2}{2}\right) \\ &= \left(\tfrac{3+(-1)}{2}, \tfrac{4+(-4)}{2}\right) \\ &= (1,0) \end{aligned}$$

$$\begin{aligned} Length &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(3-(-1))^2+(4-(-4))^2} \\ &= \sqrt{64+16} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

2)

$$\begin{aligned} (4-3i)^2 &= 16-12i-12-+9i^2 \\ &= 16-9-24i \\ &= 7-24i \end{aligned}$$

3) Complete the square. The vertex is the max.

$$\begin{aligned} -x^2+6x-7 &= -[x^2-6x+7] \\ &= -[x^2-2(3x)+9-2] \\ &= -[(x-3)^2-2] \end{aligned}$$

$$Vertex = (3,2)$$

4)

$$\begin{aligned} 0 &= f(4) \\ 0 &= 4^2+4k-9 \\ 0 &= 7+4k \\ -7 &= 4k \\ k &= -7/4 \end{aligned}$$

5)

$$\begin{aligned} f(2+3i) &= (2+3i)^2(2+3i-7)^2 \\ &= (2+3i)^2(-5+3i)^2 \\ &= (4+2(6i)+9i^2)(25+2(-15i)+9i^2) \\ &= (-5+12i)(16-30i) \\ &= -80+150i+192i-360i^2 \\ &= 280+342i \end{aligned}$$

6)

$$\begin{array}{r|l} x-3 & \begin{array}{rrr} & 4x^2+12x & +34 \\ \hline 4x^3 & -2x & +5 \\ -4x^3+12x^2 & & \\ \hline & 12x^2 & -2x \\ & -12x^2+36x & \\ \hline & & 34x \quad +5 \\ & & -34x+102 \\ \hline & & 107 \end{array} \end{array}$$

The remainder is 107.

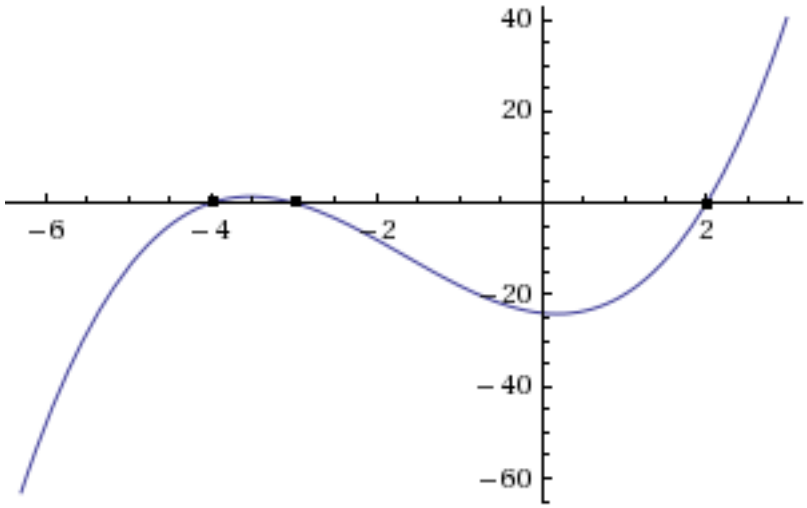
7)

$$\begin{array}{r|cccc} & 1 & 5 & -2 & -24 \\ -3 & & -3 & -6 & 24 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

So, -3 is a root and $(x+3)$ is a factor.

$$\begin{aligned} f(x) &= x^3 + 5x^2 - 2x - 24 \\ &= (x+3)(x^2 + 2x - 8) \\ &= (x+3)(x+4)(x-2) \end{aligned}$$

The function has zeros at $x = 2, -3, -4$ and $f(0) = -24$.



(x from -6 to 2)

8)

$$\begin{aligned} g(x) &= (x+3)(x-(2-3i))(x-(2+3i)) \\ &= x^3 - x^2 + x + 39 \end{aligned}$$

9)

$$\frac{x-2}{(x-3)(x+2)}$$

Domain=$\{x \,\epsilon\, \mathbb{R} \mid x \neq 3, -2\}$

Range=$\mathbb{R}$

10)

$$\begin{aligned} f(g(x)) &= f(x^3-1) \\ &= (x^3-1)^2-5 \\ &= x^6-2x^3+1-5 \\ &= x^6-2x^3-4 \end{aligned}$$

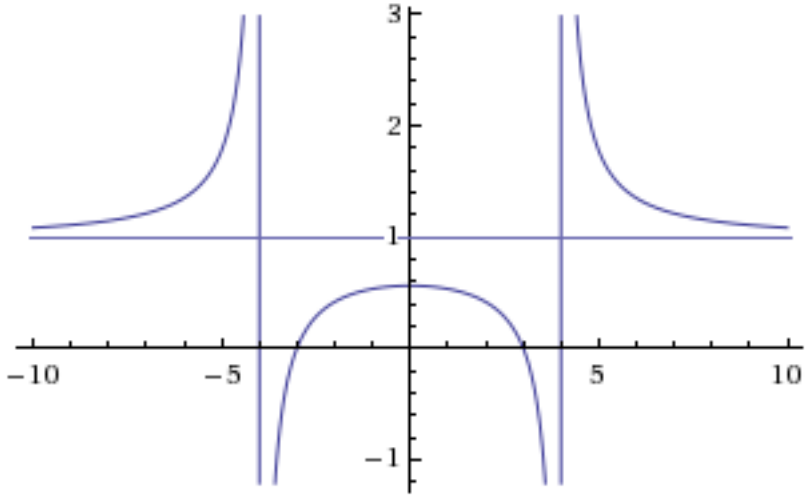
11)

$$\begin{aligned} e^{lnx^5} &= e^{12} \\ x^5 &= e^{12} \\ (x^5)^{1/5} &= (e^{12})^{1/5} \\ x &= e^{12/5} \end{aligned}$$

12)

$$f(x) = \frac{(x+3)(x-3)}{(x+4)(x-4)}$$

x-intercepts at (3,0), (-3,0)
$x \neq \pm 4 \Rightarrow$ vertical asymptotes at $x = 4, x = -4$
degree of numerator = degree of denominator = 1, so horizontal asymptote at $y = 1$



3 Trigonometry

1)

$$\begin{aligned}\cot(x+\pi) &= \frac{\cos(x+\pi)}{\sin(x+\pi)} \\ &= \frac{\cos(x)\cos(\pi)-\sin(x)\sin(\pi)}{\sin(x)\cos(\pi)+\cos(x)\sin(\pi)} \\ &= \frac{-\cos(x)}{-\sin(x)} \\ &= \cot(x)\end{aligned}$$

2)

$$g(x) = A\sin\left(\frac{2\pi}{p}[(x-4)-2]\right)+3 = A\sin\left(\frac{2\pi}{p}[x-6]\right)+3$$

3)

$$\frac{a^4-b^4}{a^2-b^2} = \frac{(a^2-b^2)(a^2+b^2)}{a^2-b^2} = a^2+b^2$$

$$\frac{\sin^4\theta-\cos^4\theta}{\sin^2\theta-\cos^2\theta} = \sin^2\theta+\cos^2\theta = 1$$

4)

$$0 < \alpha < \pi/2$$

and

$$\sin\ \alpha = 1/2 \quad \text{so}$$

$$\alpha = \pi/6$$
$$2\alpha = \pi/3$$
$$\sin(2\alpha) = \sqrt{3}/2$$

5) Use the Law of Cosines. $c^2 = a^2 + b^2 - 2ab\cos\gamma \quad \Rightarrow$

$$\begin{aligned}\cos\gamma &= \frac{a^2+b^2-c^2}{2ab} \\ &= \frac{9+36-49}{36} \\ &= -1/9 \\ \gamma &= \cos^{-1}(-1/9) \\ \gamma &\cong 96.4^\circ\end{aligned}$$

6) Show $\sin x \cos x \tan x = 1 - \cos^2 x$

$$\begin{aligned}\text{LHS (left hand side)} &= \sin x \cos x \frac{\sin x}{\cos x} \\ &= \sin^2 x \\ &= 1 - \cos^2 \\ &= \text{RHS}\end{aligned}$$

7)

$$\cos(x - \pi/2) = \cos x \cos(\pi/2) + \sin x \sin(\pi/2) = \sin x$$

8)

$$3\left(-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)\right) = 3(\cos a - \cos b)$$

4 Miscellaneous

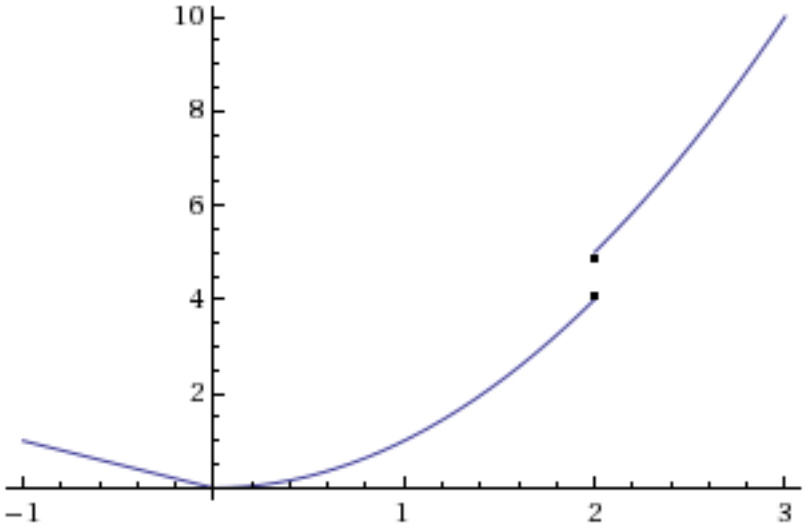
1)

$$lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)} = lim_{x\to 2}\, x + 2 = 4$$

2)

$$\begin{aligned} & \tfrac{1}{3}\left[1+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\ldots\right] \\ = \; & \tfrac{1}{3}\left[\sum_{k=0}^{\infty}\left(\tfrac{1}{2}^{k}\right)\right] \\ = \; & \tfrac{1}{3}\left[\tfrac{1}{1-1/2}\right] \\ = \; & \tfrac{2}{3} \end{aligned}$$

3) Function is continuous at $x = 0$. It has a jump at $x = 2$, therefore it is discontinous at $x = 2$.



4)

$$\begin{aligned} \text{Volume of a sphere} &= 4\pi r^3/3 \\ \text{The fraction} &= \frac{V_b - V_a}{V_c - V_a} \\ &= \frac{4\pi}{3}\left(\frac{b^3 - a^3}{c^3 - a^3}\right) \end{aligned}$$