Advanced Placement Calculus BC

AP Calculus BC course covers topics typically found in a first-year college Calculus I and Calculus II course and advances the student's understanding of concepts normally covered in high school Calculus. Major themes include differential and integral calculus. Topics include functions, graphs, limits and continuity, derivatives of basic functions, applications of the derivative, implicit differentiation, curve sketching, related rates, implicit differentiation to find the derivative of an inverse function, integration, applications of integration, geometric interpretation of differential equations via slope fields, L'Hopital's rule, techniques of integration, applications of integral calculus, logarithmic and exponential functions, parametric equations and polar coordinates, sequences and series, differential equations, and introduction to vector calculus and geometry. This course prepares students to take the Advanced Placement (AP) Calculus BC exam.
Recommended School Credit: 2.0 credit

The Basics
Prerequisites for Calculus: Precalculus Review
Average rate of change
Elementary functions and graphs
Function arithmetic and composition

Limits
Concept of limit
Finding limit graphically and numerically
Properties of limits
Intermediate theorem
Squeeze theorem

Introduction to Derivative
Concepts and types of continuity and discontinuity
Average and instantaneous rate of change
Definition of derivative
Computing derivative function from the definition
Local linearity
Derivative at a given point
Secant and tangent line at a given point
Derivative as a rate of change
Estimating the average and instantaneous rate of change from data and it graphs
Motion, position, velocity and acceleration functions
Differentiability and continuity
Power rule
Product rule
Quotient rule
Chain rule

Special Functions
Trigonometric function, exponential functions, and logarithmic functions
Graphs of trigonometric function, exponential functions, and logarithmic functions
Derivatives of trigonometric Functions, exponential functions, and logarithmic functions
Derivative of $\ln x$ and e^x

Implicit Differentiation and Calculus of Inverse Function
Finding derivative implicitly
Differentiation notation
Inverse functions: exponential and logarithmic functions
Calculus of Inverse functions
Inverse trigonometric functions
Calculus of Inverse trigonometric functions
Calculus of Hyperbolic functions
Comparing graphs of f and f'

Practical Application of the Derivative
Position, velocity, and acceleration
Critical points
Finding extrema and increasing and decreasing behavior of a curve
Local and absolute extreme values
Higher derivative and linear approximation
Newton's method
Related rates
Optimization

Curve Sketching
Finding critical points, local minimum and maximum on a curve
Comparing the graphs of f, f' and f''
The first and second derivative tests
Concavity and inflection points
Estimating derivative from data and graphs
Asymptotes and infinite limits
Intermediate value theorem
Mean value theorem
Rolle's theorem

The Basics of Integration
Antidifferentiation
Antiderivatives of Powers of x
Antiderivatives of Trigonometric and Exponential Functions
Undoing the Chain Rule
Integrating Polynomials by Substitution
Integrating Composite Trigonometric Functions by Substitution
Integrating Composite Exponential and Rational Functions by Substitution
More Integrating Trigonometric Functions by Substitution
Choosing Effective Function Decompositions
Approximating Areas of Plane Regions
Areas, Riemann Sums, and Definite Integrals
The Fundamental Theorem of Calculus, Part II
Illustrating the Fundamental Theorem of Calculus
Evaluating Definite Integrals
An Introduction to the Integral Table
Deriving the Trapezoidal Rule
An Example of the Trapezoidal Rule

Applications of Integration
Motion of a particle: net and total distance traveled
Gravity and vertical motion
Area bounded by curves
Average value of a function
Volume using cross-section

Disk method
Washer method
Shell method
Arch length
Work and Hooke's Law
Moments and center of mass

Differential Equations: Part I
Separable differential equations
Direction fields
Growth and decay problems
Logistic growth
Exponential growth and radioactive decay

L'Hopital's Rule and Improper Integral
Indeterminate forms: $\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 1^{\infty}, 0^{\infty}, \infty^{0}\right)$
Use of L'Hopital's rule
Type of improper integrals
Infinite limits of integration, convergence and divergence

The Close of Calculus I
An Introduction to Paradoxes
Paradoxes and Air Safety
Newcomb's Paradox
Zeno's Paradox
Fibonacci Numbers
The Golden Ratio

Introduction to Calculus II
Concluding differential calculus
Introduction to second half of calculus BC: integral calculus

Techniques of Integration
Integration using tables
U-substitution
Integrals involving even and odd powers and trigonometry
Integral involving partial fraction decomposition
Integration by parts
Trigonometric substitution
Integral of inverse trigonometric functions

Parametric Equation and Polar Coordinates
Calculus of parametric functions
Arc length of parmeterized curve
Polar coordinate system
Converting between polar and Cartesian form
Polar function and instantaneous slope at a given point
Area of a region bounded by polar curves

Sequences and Series
The limit of sequence
Monotonic and bounded sequences
Geometric series
Telescoping series

Test for convergent or divergent: n^{th} -term test, direct comparison test, ratio test, integral test, limit comparison test, root test, and alternating series test.
P-Series
Estimating the sum of an alternating series and the error bound
Polynomial approximations
Higher degree approximations
Taylor and Maclauring Polynomials and Series
Lagrange Formula for the Remainder
Remainder of a Taylor polynomial
Convergence of Taylor Series
Power series
Interval and radius of convergence
Integrating functions using power series

Differential Equations: Part II
Solving homogeneous differential equations
Separating homogeneous differential equations and change of variables
Solving first-order linear differential equation
Using integrating factors
Using Euler's Method of approximation
Accuracy of Euler's Method

Vector Calculus and Geometry of R^2 and R^3 (4 days)
Coordinate geometry in three dimensional space
Vectors in R^2 and R^3
Angles between vectors
Dot product
Cross product
Smooth curves of vector functions
Derivative of vector functions