

## Probability and Game Theory (GAME) CTY Sample Syllabus

**Text:**

*Game Theory and Strategy*, Straffin

**Course Schedule:**

DAY	TOPIC/ OBJECTIVES	ACTIVITIES
Day 1 (Morning)	Class overview	Discussion
	Getting acquainted	“Somebody who...” worksheet
	Class rules and procedures	Discussion
	Pre-Assessment	
	Introduction to probability	Yahtzee (play game; worksheet). More formal lecture (with participation) on the main ideas, definitions and formulas
Day 1 (Afternoon)	Introduction to probability (continued)	Lecture with participation: theoretical versus empirical versus subjective probability, and examples. Discuss Yahtzee worksheet.
	Lotteries and expected value	Simulations interspersed with discussion and calculation of the expected value of a lottery ticket
Day 1 (Evening)	Probability practice	Worksheet and challenge problems
	Introduction to zero-sum games	Students play “A Fascinating Game” then read Ch. 1 in Straffin

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Day 2 (Morning)	Probability	Students present harder worksheet problems from preceding night at the blackboard
	Two-person zero-sum games	Discussion and analysis of “A Fascinating Game”; students create and play their own matrix games
	Mixed strategies (Matching Pennies)	Play Matching Pennies with a partner; class discussion
Day 2 (Afternoon)	Mixed strategies	Intuitive analysis of a game with a mixed strategy, then play the game.
	Permutations and combinations (continued)	Lecture with participation, hands-on activities on number of ways of arranging items
	Mixed strategies	Calculate mixed strategies for games studied so far, first using algebraic approach, then with oddments
Day 2 (Evening)	Reinforcement of 2x2 games	Solving 2x2 games worksheet
	Article preparation	Students prepare articles found in computer lab
	Lottery worksheet	Worksheet on probabilities and expected values in the lottery
	Critical thinking regarding strategic moves	America’s Cup example: introduce the scenario; describe what happened; discuss what the American boat should have done (collect written suggestions from students)
	Larger games	Students play 4x4 game and record results for discussion tomorrow

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Day 3 (Morning)	Analyzing games with more than two strategies	Discussion of game from Day 2 evening session; generalization of 2x2 game methods to larger games
	Empirical probability	Students blindly draw poker chips (blue and white) repeatedly from a bag, guess the composition of the bag
	Solving 2xn and nx2 games using ladder diagrams	Individual and group examples
	Critical thinking regarding strategy	“Cab ride in Israel” story. Describe the situation, and then discuss what the parties (cab driver, riders) should have done.
Day 3 (Afternoon)	Mixed strategies in 3x3 games	Class rock-paper-scissors tournament, followed by discussion of optimal mixed strategy and when/whether to use it
	Polling and confidence intervals	Read news article about presidential election poll in Alaska; discuss formula and meaning of 95% confidence interval
	Jamaican fishing example	Application of game theory to anthropology; students calculate the optimal strategies for fishermen and current, then discuss the appropriateness of using game theory in this situation
Day 3 (Evening)	Reinforcement of zero-sum matrix games	Read chapters in Straffin on dominance, movement diagrams and mixed strategies, then do a few problems from Straffin.
	Reinforcement of permutations and combinations as applied to probability	Problem sheet

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	Introduction to Games against Nature: umbrella problem	Worksheet on different ways of looking at the problem of whether to carry an umbrella.
Day 4 (Morning)	Review of matrix games	Students present more difficult problems from previous evening session
	Mixed Strategies and Unpredictability	Discussion of importance of unpredictability, real-world applications of mixed strategies. <i>Princess Bride</i> clip on trying to “psych out” your opponent or modify the rules of a game without his knowledge
	Extra time for leftover Evening 3 work	Students do work from preceding evening
	Strategic thinking; working backwards	Other Person’s Index Card Is Always Greener: play game; discuss strategy and expected value approach; play again, then discuss the rational approach
	Newcomb’s Problem and Free Will: Introduction	Discussion on Newcomb’s scenario: two boxes and an omniscient “Being” (Chuck Norris). Box #1 has \$1,000 in it, and box #2 may have \$0 or \$1,000,000. You may take Box #2 or both boxes, but the being puts \$1,000,000 in Box #2 if and only if you take Box #2 only. Discuss arguments for both possible actions
Day 4 (Afternoon)	Newcomb’s Problem and Free Will: Further Development	Continuation of morning activity
	Conditional probability: the Monty Hall problem	Demonstrate game for the class; collect data by simulation; discuss resolution of the problem with students or instructor presenting multiple different resolutions

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	Bayes' Rule	Present Bayes' Rule and have students apply it to different scenarios
	Journal writing	Students write a paragraph or two on what topics they found most interesting, challenging or confusing so far in class
Day 4 (Evening)	Reinforcement	Students calculate probabilities of various poker hands (probability and combinatorics reinforcement). Straffin exercises on Newcomb's Problem. Worksheet on Bayes' Rule
	Bayes' Rule applet	Students use Bayes' Rule applet to solve probability problems. When finished, students begin selecting topics for term projects
	Independent research	Students begin selecting topics for final group project
Day 5 (Morning)	Review of evening problems from Day 4	Students present solutions to more difficult combinatorics/probability problems from evening session of Day 4
	Student article presentations	Three students present articles on game theory previously researched in the computer lab.
	Game trees and sequential games	As a class, analyze a game tree for a game with perfect information and sequential moves
	Game trees and sequential games: games of imperfect information	Students play a very simplified version of poker, then analyze the game tree, expressing the game in extensive and normal form and analyzing the results

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	Games against Nature	Review umbrella worksheet from Day 3. Do another Game against Nature, solving using probabilities and expected value. Discuss different approaches when the probabilities are unknown, which approaches students prefer and why.
Day 5 (Afternoon)	Games against Nature (continued)	Continuation of morning discussion, with a few additional approaches added
	Applications of Probability: gambling	Lecture with participation on relative payoffs of casino games
	Roulette story	Present roulette scenario, then ask students how the economist involved should have bet
Day 5 (Evening)	Game trees homework	Problems from Straffin (p. 43, #4, 5; read Ch. 8, then do p. 47 #2)
	Independent research	Students work on term project in computer lab (may work in pairs if desired)
Day 6 (Morning)	Introduction to non-zero-sum games	Play non-zero-sum game, discuss the outcome, and contrast with zero-sum games. Analyze game using movement diagram and dominance. Discuss real-life non-zero-sum games, and then play another one.
	Introduction to Nash equilibrium	Students discuss how we could generalize mixed strategies encountered in zero-sum games. Introduce idea of mixed Nash equilibria. Discussion of Pareto optimality and SSS games, with examples

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	Utility theory	Activity to demonstrate how students could determine the relative utility of different outcomes, while noting the arbitrariness of utility scales.
Day 6 (Afternoon)	Utility theory (continued)	Example of two RAs who have different preferences for what song to play at the dance; they decide to assign each song a utility and add utilities, then find themselves in a situation of each trying to one-up the other by changing the utility scales. Example where the instructor's preferences are not transitive, and the TA takes advantage of this in a somewhat comical way
	A non-zero-sum game: Battle of the Sexes	Hand out description of game; students (in groups) make payoff matrix and discuss Nash Equilibria, Pareto optimality and whether game is SSS, then discuss as a larger group
	Non-zero-sum game: Chicken	Premise introduced. Students write out possible payoff matrices and fill out game outline. Discussion of real-world examples and differences between Chicken and previously studied games.
	Non-zero-sum game: Stag Hunt	Premise introduced. Students come up with payoff matrix and discuss the dilemma the game presents. Play a multi-player variant of Stag Hunt
Day 6 (Evening)	Practice with Games Against Nature	Reading and a problem in Straffin.
	Coordination Game: VHS vs. Beta	Worksheet
	Reinforcement of Nash Equilibria/Pareto Optimality	Reading and problems in Straffin.

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	Independent research	Continue work on term projects in computer lab
Day 7 (Morning)	Stag Hunt	Completion of previous day's activity
	Midterm review	Examples of various types of problems on topics considered so far (a combination of material prepared by the instructor and student questions, with emphasis on the latter)
	Application to biology: Evolutionarily Stable Strategies	PowerPoint presentation on ESSs, with students calculating numbers to be put in the relevant payoff matrices
Day 7 (Afternoon)	Prisoner's Dilemma simulation	Put students into pairs and interrogate them for some supposed (and ridiculous) "crime," presenting them with the Prisoner's Dilemma scenario
	Prisoner's Dilemma discussion	Review results of simulation. Students write payoff matrix. Students find the dominant strategy and any problems with it (it is not Pareto optimal), then discuss as a class and complete game outline for the game. Students come up with real world examples, and discuss these.
	Strategic moves	PowerPoint presentation on how two-player non-zero-sum games change (potentially benefiting one or both players) if players do not move simultaneously or if one player can make a commitment, threat or promise, with breaks for students to analyze game situations. Discussion of credibility of strategic moves. Careful analysis of examples
	Probability problem	Discuss the probability of getting a full house in a single Yahtzee roll

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	Journal writing	Students write a paragraph or two on what topics they found most interesting, challenging or confusing
Day 7 (Evening)	Midterm	Midterm (covers probability, combinatorics, zero-sum games)
	Prisoner's Dilemma	Homework: read Prisoner's Dilemma chapter in Straffin, then solve some problems at end of chapter and describe at least two situations that could be represented by each of Prisoner's Dilemma and Chicken.
	Trust Game	Students play game
Day 8 (Morning)	Recognition of strategic situations: <i>Thirteen Days</i>	<i>Thirteen Days</i> (movie on Cuban Missile Crisis). Discussion of applications of game theory in the movie
Day 8 (Afternoon)	Recognition of strategic situations: <i>Thirteen Days</i>	Continue discussion from the morning
	Student article presentations	Students present articles on game theory previously researched in the computer lab
	Return midterm	Return midterms, and discuss selected problems, with students presenting solutions to each other
	Application of utility theory: "Deal or No Deal"	Students discuss how they would respond to scenario from TV show, and when a player might not maximize expected payoff. Decreasing marginal utility.

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Day 8 (Evening)	Voting preparation	Students provide the names of four sodas, then each student casts a vote by ranking the four according to his/her preferences. In groups, students develop methods to combine the individual rankings into class rankings
	Strategic moves homework	Read Strategic Moves chapter in Straffin, then solve five games given in Straffin to determine which strategic moves, if any, would lead Rose to get a higher payoff.
	Independent research	Continue work on term projects in computer lab.
Day 9 (Morning)	3-person games	Students discuss how to represent a 3-person matrix game, play the game several times, find the equilibria on a worksheet, then play again
	Coalitions in 3-person games	Worksheet followed by discussion: consideration of when and how players in the 3-player game would form a coalition and calculations of payoffs to different players depending on coalition formed
	4-player games with coalitions	Each group of students is given \$13 in poker chips if at least three out of four of them can agree on how to split them and explain their method
	Voting: introduction	Each group from preceding evening explains its voting procedure, and use these on the soda voting. Presentation of different voting schemes (including those that students suggest), asking students to apply each to a few preference schedules. As schemes are suggested, students apply the schemes to preference schedules and identify weaknesses in the schemes

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Day 9 (Afternoon)	Voting: Arrow's Theorem	Continuation of voting introduction and presentation of Arrow's Theorem. Discussion of the relative strengths of the different systems.
	Application of game trees and strategic moves: Paradox of Benevolent Authority	Group work on a variant of Prisoner's Dilemma in which there is a third player who wishes to maximize the first two players' payoffs and how such maximization could be implemented
	Student article presentations	Students present articles on game theory previously researched in the computer lab.
Day 9 (Evening)	Reinforcement of voting material	Lots of voting problems, with students who finish early doing extra problems from chapters previously covered in Straffin or further voting problems requiring more creativity
	Independent research	Work on projects
Day 10 (Morning)	Fair Division: Divider-Chooser	Pairs of students devise method to split up a candy bar fairly. Discussion of student proposals and Divider-Chooser method
	Fair Division: Lone Divider	Define fair shares, fair division and fair division schemes. Can students come up with an extension of Divider-Chooser to three players? Present Lone Divider method for three players
	Tragedy of the Commons: Pimsler's Dementia	Divide class into groups and have them play, with no communication, and keep track of the results. Discuss results and strategies, then play again and continue discussion.

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	Tragedy of the Commons: more general introduction	Define Tragedy of the Commons, and discuss its relationship to Prisoner's Dilemma.
	Tragedy of the Commons: Bankruptcy example	Group worksheet
Day 10 (Afternoon)	Tragedy of the Commons: Bankruptcy example (continued)	Group worksheet, then discussion of more challenging problems
	Variant on the Prisoner's Dilemma: OPEC example	Worksheet on OPEC countries cheating or not cheating; discussion and comparison to Prisoner's Dilemma
	Application of probability: Birthday Problem	Discussion interspersed with calculation on the Birthday Problem
Day 10 (Evening)	N-Player Games: Reading and Problems	Reading and a problem from Straffin chapter on N-player games. (Those who finish can do more voting problems.)
Day 10 (Evening)	Independent research	Continue work on projects.
Day 11 (Morning)	Arrow's conditions review	Review of some common pitfalls in using Arrow's conditions and how to avoid them
	Student project presentations	Some students present their projects now.
	Fair Division: Last Diminisher	Describe method for dividing a cake or other item (such as an island drawn on a transparency). Groups of five or six students work through examples and discuss fairness of the method
	Fair Division: Envy-Free Methods	Discussion: Which methods studied so far are envy-free, and why?

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	Tragedy of the Commons: Pimsler's Dementia (continued)	Play Pimsler's Dementia with the other GAME section, and discuss results
	Fair Division: Sealed Bids for discrete items	Simulation of division of an estate among multiple heirs, and discussion of the method's weaknesses
Day 11 (Afternoon)	Student project presentations	Some students present their projects now.
	Auctions: sealed bids	Auction off a few items by sealed bids (or variants) and discuss fairness of the methods. Students do Strategy of Auctions worksheet and begin to discuss it
	Fair Division: Last Diminisher	Divide a cake using the Last Diminisher method.
Day 11 (Evening)	Reinforcement of fair division	Problems from Tannenbaum book.
	Voting, Negotiation and Strategy: Coalitions game	Students play Coalitions for 30 minutes.
Day 12 (Morning)	Student project presentations	Some students present their projects now.
	Auctions: low-bid sealed bids auctions	Strategy of Auctions worksheet continues. Discuss results, including calculation of optimal bid if opponents bid honestly.
	Auctions: English Auctions and the Winner's Curse	Auction off candy, then bags of unknown quantities of poker chips using English Auction. Discussion of the Winner's Curse, in small groups then as a class

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	Strategic Voting: voting insincerely	Review voting methods, and introduce the idea of insincere or strategic voting. Give fictionalized preference schedules from 2000 presidential election (Bush vs. Gore vs. Nader), and have discussion of who might vote strategically and why. Students work through example of strategic voting in a runoff scenario.
	Strategic Voting: sequential voting	Whimsical example of a CTY student accused of a terrible crime. Three of the SRAs will decide between execution, life in prison or acquittal. Students determine the outcome depending on the order in which the possibilities are considered.
Day 12 (Afternoon)	Student project presentations	Some students present their projects now.
	Voting power: Banzhaf power index	Define notation for weighted voting and work through several examples. Define Banzhaf power index and do an example as a class, then more as small groups.
	Auctions: sealed bids	Auction off a few items by sealed bids (or variants) and discuss fairness of the methods
Day 12 (Evening)	Apportionment	Groups come up with an apportionment method that will always lead to exactly the total number of seats in the legislature that are desired.
	Voting power reinforcement	Problems from Tannenbaum.
	Strategic thinking: three-way duel	Students (first individually, then in pairs, then as a class) determine what the first shooter's best strategy is, and why

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	Auctions and weakness of the Nash Equilibrium	Article on differences between how people are predicted to behave by Nash Equilibrium and how they actually behave. Discussion of limitations of Nash Equilibrium
Day 13 (Morning)	Auctions: Chinese Auction	Run Chinese Auction. Discuss strategy and Winner's Curse.
	Voting power: Banzhaf power index	Partly as a class and partly in groups, solve the example that led to Banzhaf's lawsuit
	Auctions: Dutch Auction	Hold Dutch auctions, then discuss strategy.
	Auctions: wrap-up	Discuss overall lessons learned from study of auctions.
	Final exam review	Remind students of main subjects of course; give students lots of problems to work through; instructor and TA answer questions
Day 13 (Afternoon)	Final exam review	Review as a class the more difficult problems from the morning
	Apportionment	Give historical background on apportionment, then each group describes its apportionment method. Discuss advantages and disadvantages, and define terms. Students work through example where the Alabama Paradox occurs and identify why it is paradoxical.
Day 13 (Evening)	Final exam	Final exam
Day 14 (Morning)	Pointing game	Students play pointing game
	<i>A Brilliant Madness</i>	Students watch documentary on John Nash, then discuss

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	Return final	Review a few of the more difficult problems
	SPEs	Student Program Evaluations
Day 14 (Afternoon)	People v. Collins	Students analyze situation in which couple were convicted almost solely on the basis of probabilistic evidence, then discuss
	Rock Paper Scissors tournament	Second Rock Paper Scissors tournament, with a few moves added to the conventional ones, followed by derivation of the optimal mixed strategy in the modified game
	“Mail Order Prophet”	Watch “Alfred Hitchcock Presents” episode in which a criminal executes a clever scam based on probability
Day 14 (Evening)	Casino games	Each student is given choice between two dice games: Krume and Slerm (with the option not to play either). Students invent casino games to play; the instructor and TA (the house) play game against any student who wishes to play, as long as the game has an expected payoff of at most 1 to player
	Auction	Auction off a few items using varying auction methods
Day 15 (Morning)	St. Petersburg Paradox	Auction off a chance to play the St. Petersburg game, then find its expected value (infinite)
	Final auction	Using the poker chips students have accumulated during the session, auction off various small prizes using various auction methods covered in class.

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	Conclusion	Summarize the main points of the course, focusing less on mechanical computations (e.g. oddments) and more on key ideas (think strategically, considering that other people, whose interests may or may not be aligned with yours, are also thinking strategically).